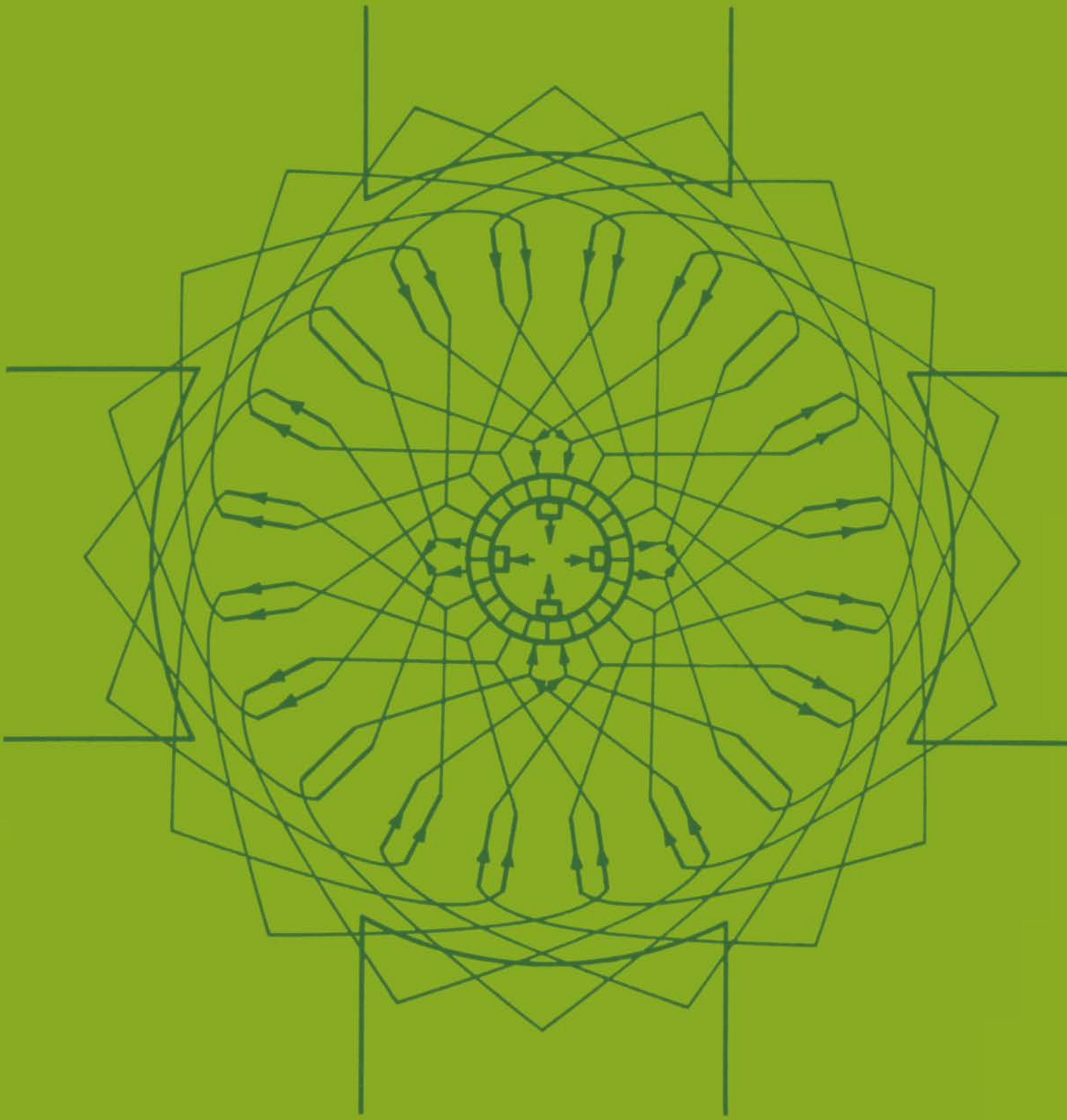
INTRODUCTION TO ELECTRICAL MACHINES



A. R. Daniels

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Preface

The attention given to the study of the performance of electrical machines has, in recent years, been reduced in many higher-education courses. Developments have taken place which, in the early stages, emphasize the similarities, rather than the differences, between the various machine types. In preparing this text, a basic approach to the formation of operating equations has been made using the concept of a so-called 'general machine' whose operation covers all the well-established different machine types. In this manner, the concepts of synchronous and asynchronous operation and the action of both a.c. and d.c. commutator motors are introduced at the earliest possible stage. Since the action of all electrical machines depends on energy storage in a coupling field, a simplified treatment of electromechanical energy conversion has been included.

Once the common principles governing the operation of all forms of electrical machine have been established, each different machine type is dealt with separately in a conventional manner. Modern teaching courses take place at a rapid pace and the salient facts in the basic analysis of each machine type are considered first in each chapter. In this way, more advanced topics in certain chapters can be omitted without disrupting the continuity of the text. The first section of the book (chapters 1–3) considers general principles and also deals with winding arrangements. The performance of the transformer and the various machine types is considered in the next section (chapters 4–8) and the final section (chapter 9) deals with the characteristics and application of the power thyristor when used to control the speed of both d.c. and a.c. motors.

Throughout the text, distinction between phasor and scalor quantities is made by the use of bold type to designate phasors. In the particular case of phasor diagrams which, in any case, always depict phasors, bold type is not used. A simple circuit convention for a source of energy (generator) and for a sink of energy (motor) is used throughout the text. This particular convention is only one of many possible conventions and its only requirement is that it should be correctly and continuously applied.

I would like to thank the Council of the University of Bath for permission to use past examination papers. Any dedication made must be to my past students, all of whom survived after treatment with the material contained in this textbook.

1. The basis of operation of electromagnetic machines

1.1. Basic elements

All electrical machines are forms of energy converter and the vast majority are electrical–mechanical, arranged to convert from electrical to mechanical energy (motors or sinks) or from mechanical to electrical energy (generators or sources). There will always be some form of coupling field between the electrical and mechanical systems and, in most cases, this takes the form of a magnetic field. The structure on which the field circuit is located may be either stationary or rotating, depending on the particular form of machine. In many cases the field will be electromagnetic and field coils carrying the field current will be wound on a magnetic structure. The iron forming this structure will be laminated in order to reduce the field iron losses if the field current is alternating or contains an alternating component. Two different forms of construction are used for the field circuit. Firstly the so-called 'salient-pole' arrangement can be used, in which field coils are concentrated and wound around protruding poles. This form of construction is only used for machines with direct-current (d.c.) field supplies. The second form of construction is that in which the field coils are distributed in slots cut into a cylindrical magnetic structure, and this arrangement is commonly used on certain forms of alternating current (a.c.) generators. It is important to note at this stage that the field structure can, in general, be physically situated on either the stationary member (stator) or rotating member (rotor) on the machine.

The action of all forms of electromagnetic machine is based on two fundamental laws. Firstly Faraday's law, which states that, when there is relative motion between the magnetic field and a coil situated in that field, there will be an e.m.f. induced in that coil, whose magnitude is equal to the time rate of change of flux linkage between the field and the coil. The second law is based on an observation by Maxwell which states that, if a current-carrying conductor is placed in a magnetic field, a force will be exerted and relative motion will take place between the structure carrying the conductor and that carrying the field. There will be two electrical members of an electrical machine: the field and the so-called armature in which work is done. In general, the electrical circuit representing the armature of the machine will consist of coils distributed in slots cut into a cylindrical magnetic structure. The armature currents will always be alternating and, in these circumstances, the iron forming the magnetic circuit of the armature will be laminated in order to reduce the associated iron losses. When there is current in the armature coils, an m.m.f., known as the m.m.f. of armature reaction, will be set up. There will be a magnetic flux associated with this m.m.f., which will combine with the field flux to produce the resultant total flux in the machine. If it is assumed that the two component fluxes, those of field flux and armature reaction flux, exist separately, then the speed of rotation of these two component fluxes in space must be the same if a steady electromagnetic torque is to be produced.

In general, either the field or the armature can physically rotate and, for example, when the armature of the machine is on the rotor, the machine is known as a rotating armature machine. It is important to note that the basic laws stated previously explicitly govern the operation of *all* forms of electromagnetic machinery. The form of different rotating machines (a.c. or d.c., shunt or series, etc.) is governed by external constraints such as the form of electrical supply connected to the field circuit or the mechanical method of connection to the rotating member.

Mechanically the main components of an electrical machine consist of a stationary element, known as the stator, and a rotating element, known as the rotor, with an air-gap between them. Special mechanical arrangements must be provided when electrical connections are to be made to the rotating member. Such connections are normally made through carbon brushes bearing on either a so-called slip-ring or a commutator, rotating with the rotor. A slip-ring is a continuous ring, usually made of brass, and there will only be one electrical connection to each slip-ring. A commutator is, in effect, an elegant mechanical switch and consists of hand-drawn copper segments separated and insulated from each other by mica. A simplified schematic diagram showing a typical commutator together with the winding connections to the commutator is given in Fig. 1.1 in both radial and developed form. The winding considered in Fig. 1.1 is known as a double-layer winding and consists, basically, of a number of interconnected coils. Each coil has two sides and in a double-layer winding there are two separate coil sides in each slot cut into the cylindrical magnetic structure. The particular rotor shown is cylindrical and has 16 slots carrying a total of 16 coils. The pitch of the coil in this case is 180 mechanical degrees (°m) so that a coil is formed between the 'top' of slot 1 and the 'bottom' of slot 9 and so on. This arrangement is more readily understood by inspection of the developed winding diagram of Fig. 1.1(b). This type of winding is known as a distributed, double-layer, lap connected winding and a useful schematic representation of this arrangement is given in Fig. 1.1(c).

Many forms of electrical machine operate from a 3-phase a.c. supply and the armature winding of such a machine will consist of a 3-phase distributed winding for which the three separate phase windings (a, b, and c) are wound in an identical manner. In, for example, the particular case of the 4-pole machine, 360° mechanical must correspond to 720° electrical since for a full-pitch coil, one pole-pitch must always equal 180°e. Then, in general, for a machine with p poles:

$$360°\text{m} = \frac{p}{2}\,360°\text{e}$$

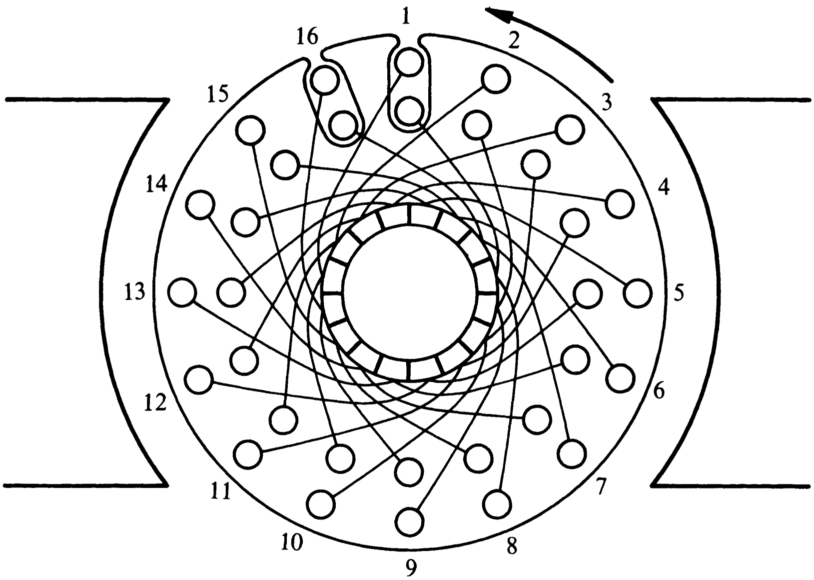


Fig. 1.1. Simplified form of commutator winding:
(a) Radial diagram showing front coil connections.

'BACK' CONNECTIONS

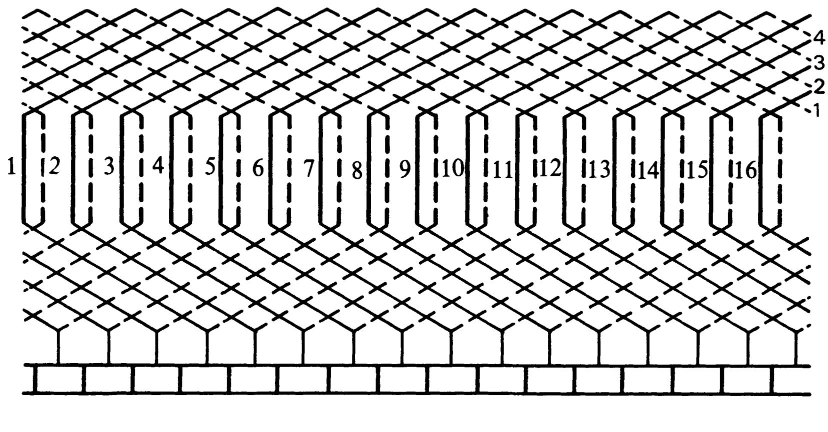


Fig. 1.1(b) Developed diagram of complete winding.

or $$\frac{p}{2} \text{ electrical radians} = 1 \text{ mechanical radian.} \tag{1.1}$$

When 3-phase voltages are applied to a 3-phase winding, an m.m.f. will be set up and the distribution of the total m.m.f. around the periphery of the stator is non-sinusoidal but the error involved in assuming a sinusoidal space distribution of m.m.f. will, in the basic analysis, be small.

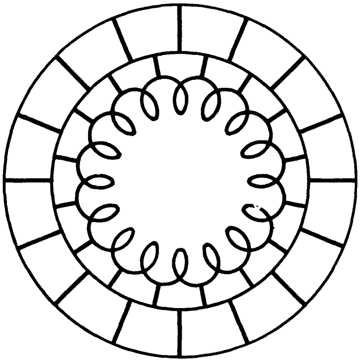


Fig. 1.1(c) Schematic representation of commutator winding.

1.2. Production of a rotating m.m.f.

In the case when the supply to the machine is a 3-phase balanced a.c. supply of frequency f_s Hz, such that the angular velocity $\omega_s = 2\pi f_s$, it is possible to produce an m.m.f. which rotates, in synchronism with the supply frequency, relative to the structure on which the coils are situated. This structure may be either the stator or rotor of the machine and the speed of rotation of this rotating m.m.f. in space will then be the algebraic sum of the speed of the m.m.f. relative to the structure and the mechanical speed of the structure itself. The stator of the machine can be assumed to be stationary in space and, in general, the rotor can be assumed to be rotating at some mechanical speed ω rad/s.

Consider a simple 2-pole, 3-phase winding arrangement of which each phase is assumed to produce a sinusoidal space distribution of m.m.f. The peak value of the three phase components of m.m.f. will each pulsate sinusoidally with time between positive and negative maximum values as the a.c. current in each phase varies sinusoidally with time. The total m.m.f. produced will be the algebraic sum of the three separate phase values at any given instant in time. The distribution of the m.m.f. of each phase and that of the total m.m.f. are shown in Figs. 1.2(a) and (b) for two different instants in time corresponding to positive maximum values of two of the phase currents, and this time interval is 120°e. It can be seen from Fig. 1.2 that, during

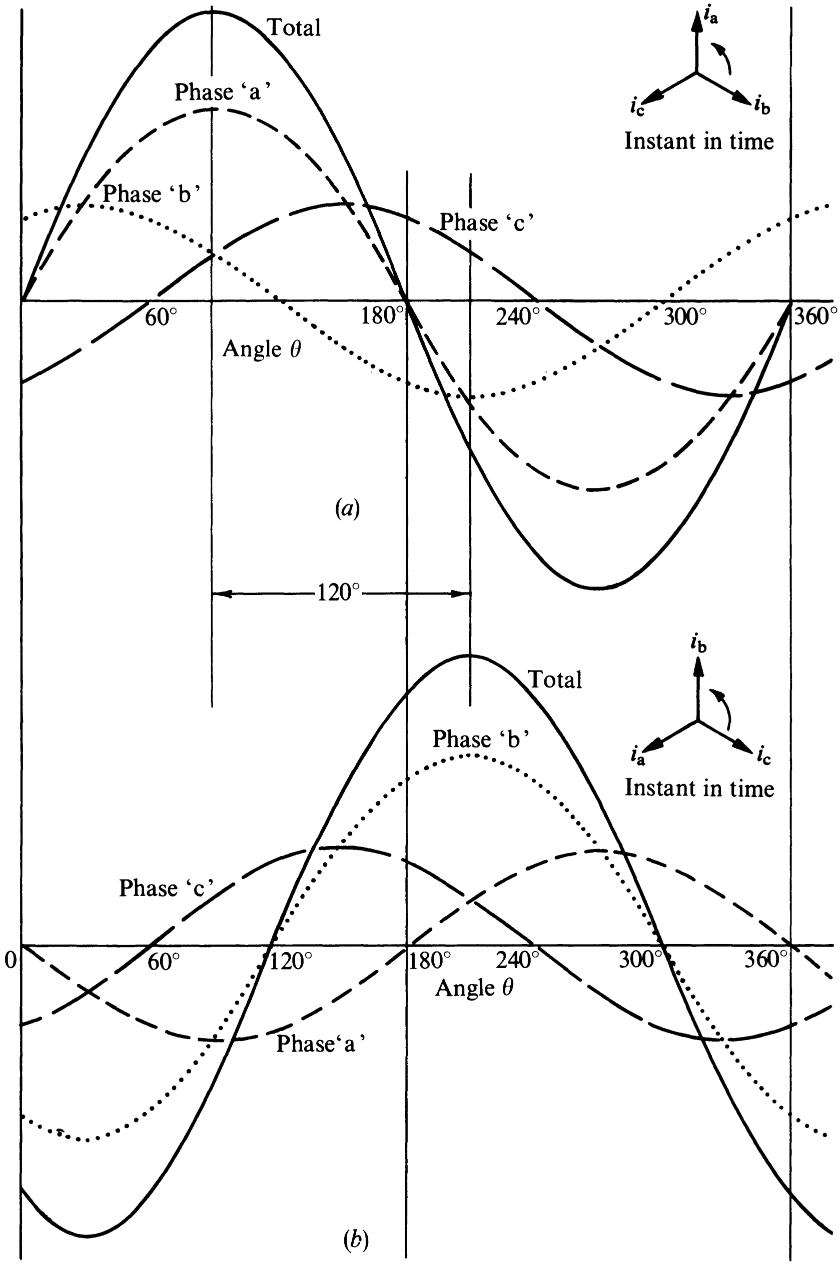


Fig. 1.2. Production of a rotating m.m.f.:
(a) Spatial m.m.f. distribution when the phase 'a' current is a maximum.
(b) Spatial m.m.f. distribution when the phase 'b' current is a maximum.

this time interval, the total waveform is sinusoidal and has rotated through 120°e of the periphery of the structure carrying the 3-phase winding, i.e. the wave of total m.m.f. rotates in synchronism with the time variations in current. It is also apparent from Fig. 1.2 that the peak amplitude of the total m.m.f. wave remains constant at 1·5 times that of the phase value. This effect results in the production of a total m.m.f. rotating in synchronism with the supply frequency and, in general, for a 3-phase machine with p poles operating from a 3-phase supply of frequency f Hz, the speed of rotation of the total m.m.f. is known as the synchronous speed, ω_s and is given by:

$$\omega_s = \frac{2\pi f}{p/2} \text{ rad/s} \tag{1.2}$$

or

$$N_s = \frac{120f}{p} \text{ r.p.m.} \tag{1.3}$$

The same result can be obtained analytically. For any point p, in Fig. 1.2 at angle θ from the origin, the total m.m.f. F is:

$$F = F_a \sin\theta + F_b \sin(\theta - 120°) + F_c \sin(\theta - 240°).$$

But $F_a = F_m \sin\omega_s t$, $F_b = F_m \sin(\omega_s t - 120°)$, $F_c = F_m \sin(\omega_s t - 240°)$. Then, on substitution and solution:

$$F = \frac{3}{2} F_m \cos(\theta - \omega_s t). \tag{1.4}$$

Equation (1.4) shows that the total m.m.f. has a constant amplitude, is a sinusoidal function of the angle θ and rotates in synchronism with the supply frequency.

The corresponding flux density B is given by

$$B = \frac{\mu_0 F}{g} \tag{1.5}$$

where g is the 'length' of the air-gap with the reluctance of the magnetic circuit neglected.

1.3. The general machine

It has previously been stated that the principles on which all electromagnetic machinery operate are the same and that different machine types are obtained by the use of different forms of supply and different mechanical elements. In these circumstances it is useful to introduce the concept of a so-called 'general machine' and to show that the different machine types are all particular cases of this general arrangement.

Consider a machine with a 3-phase, 2-pole winding on the stator and a 2-pole lap-connected winding on the rotor connected to both a commutator carrying a 3-phase arrangement of brushes, and to three slip-rings with tappings spaced at 120°e connected to the three separate slip-rings. The general arrangement is given in Fig. 1.3. When a 3-phase voltage system is applied to the stator winding, an m.m.f. rotating at synchronous speed ω_s with respect to the stator is set up. An m.m.f. rotating at some speed ω_2 with

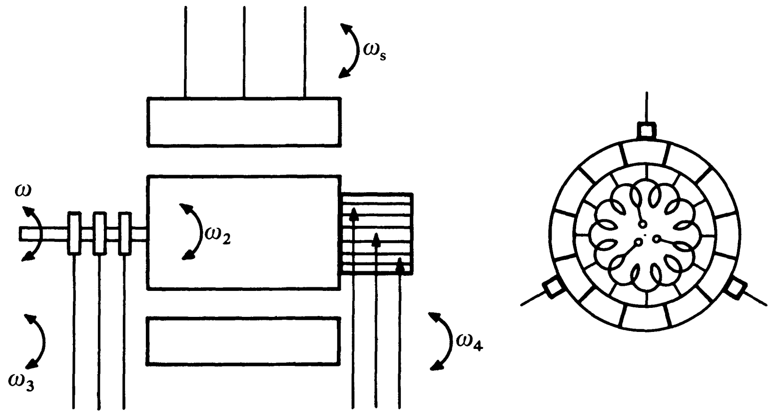


Fig. 1.3. Arrangement of the basic general machine.

respect to the rotor will be induced in the rotor winding and the rotor itself will rotate at some mechanical speed ω. Then the speed of the rotor m.m.f. in space is $\omega \pm \omega_2$ and the speed of the stator m.m.f. in space is ω_s. In order that a steady torque is produced, the two component m.m.f.s must rotate at the same speed in space, so that:

$$\omega_s = \omega \pm \omega_2. \tag{1.6}$$

Since the winding on the rotor is connected to both a commutator and slip-rings, it is reasonable to assume that alternating 3-phase voltages will appear at both. The frequency of the voltage at the slip-rings will be assumed to be f_3 Hz such that $\omega_3 = 2\pi f_3$ and that at the commutator will be assumed to be f_4 Hz such that $\omega_4 = 2\pi f_4$. The basic difference between the action of slip-rings and that of a commutator is illustrated in Fig. 1.4. The conducting coil connected to the slip-ring is always connected to the brush, regardless of the speed ω and position of the rotor. Thus the frequency of the voltage at the slip-ring brush must be equal to that of the rotor m.m.f. relative to the rotor, so that $\omega_3 \equiv \omega_2$. However, in the case of the commutator, the conducting coil is only conducting current while it is physically under the commutator brush, i.e. while it is stationary with respect to the commutator brush. It follows that the frequency of the voltage at the commutator brush must be that of the rotor m.m.f. in space, so that $\omega_4 = \omega \pm \omega_2$.

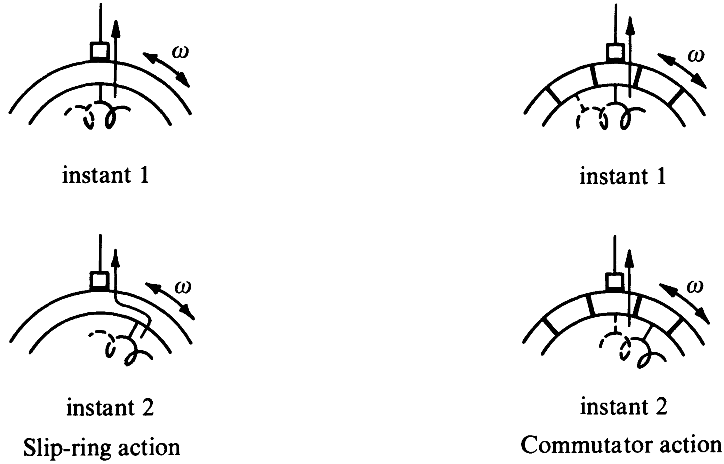


Fig. 1.4. Commutator and slip-ring frequencies.

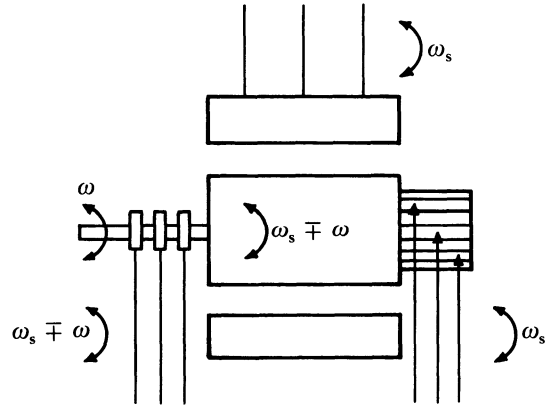


Fig. 1.5. Final arrangement of the general machine.

The arrangement of the so-called general machine can now be redrawn in the manner shown in Fig. 1.5 in which all the required frequencies have been written in terms of the mechanical speed of rotation, ω, of the rotor and the angular speed, ω_s, of the m.m.f. set up when polyphase a.c. of frequency f Hz is applied to the stator. Each of the well-known electromagnetic rotating machines can then be obtained as a particular case of this general machine.

1.4. Particular cases of the general machine

(*a*) *The synchronous machine*

The polyphase synchronous machine operates with direct current applied to the field winding, assumed to be on the rotor and supplied through two slip-rings, and polyphase a.c. supplied to the armature, assumed to be on the stator. This mode of operation is illustrated in Fig. 1.6 and the rotor m.m.f., which is obtained from a d.c. source, is stationary with respect to the rotor structure. Thus under steady-state conditions $0 = \omega_s \pm \omega$ and a synchronous machine will only operate under steady-state conditions at synchronous

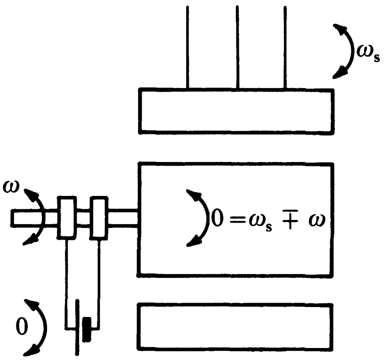


Fig. 1.6. The general machine as a 3-phase synchronous machine.

speed, as its name implies. It follows that, when the load on a synchronous machine is to be changed, the machine cannot operate at synchronous speed during the load transition. Thus in the case of a generator supplying power to a high capacity, fixed frequency system (infinite busbar) an increase in the electrical output power of the generator is brought about by increasing the power supplied by the prime mover. The speed of the rotor will then momentarily increase and the axis of the field m.m.f. will advance on the axis of the armature m.m.f. by some angle θ. This results in an increase in electrical output power and the angle θ continues to increase until the output power is equal to the nett input power. At this point, the machine will lock into synchronism and continue to rotate at synchronous speed until a further load change takes place. The angle θ is known as the load angle δ of a synchronous machine and there will be a sinusoidal relationship between power and load angle ($P \alpha \sin \delta$) for a machine with a cylindrical stator and rotor construction.

The same general argument can be applied to the operation of the synchronous motor, except that an increase in mechanical load will decrease the speed of the rotor during the transition period, so that the axis of the field m.m.f. falls behind that of the stator m.m.f. by the required value of load angle. When a synchronous motor is stationary, it is not rotating at synchronous speed and cannot, therefore, produce a steady torque. It follows immediately that a synchronous motor is not self-starting.

When the load on a synchronous machine changes, the load angle must change from one steady value to another and, during this change, oscillations in the load angle must occur. Such oscillations must be controlled and it is common practice to fit an additional short-circuited winding, known as a damper winding, on the field structure, in order to damp out these oscillations. When the machine is operating under steady-state conditions, at synchronous speed, there will be zero rate of change of flux linkage with this damper winding and, therefore, no voltage induced in it. Under these circumstances, this winding will have no effect on the steady-state performance of the synchronous machine.

(*b*) *The direct current machine*

Such a machine operates with direct current applied to the field winding which is generally situated on the salient-pole stator of the machine and direct current applied to a commutator connected to the armature winding situated on the cylindrical rotor. This mode of operation is illustrated in Fig. 1.7 and the operating conditions satisfy those of the general machine. Thus the d.c. machine will operate under steady-state conditions, whatever the rotor speed, ω, and the d.c. motor will produce starting torque. The armature current is alternating and, in this particular case, the action of the commutator is to frequency change the armature current from a frequency governed by the mechanical speed of rotation to zero frequency at the commutator brushes.

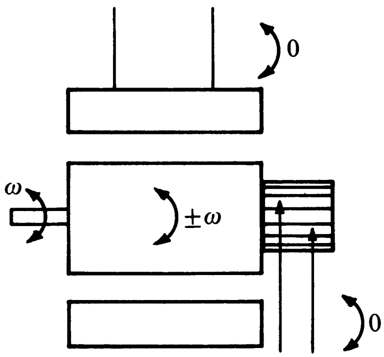


Fig. 1.7. The general machine

Since both the armature and field circuits carry direct current, they can be connected together in either series or parallel. In the particular case when the armature and field circuits are connected in parallel, the machine is known as a shunt machine and the field coils will be wound with a large number of turns and will carry a relatively small current. When these circuits are connected in series, the machine is known as a series machine and the field winding which now carries the full armature current will be wound with a small number of turns. If the machine has both a series and a shunt field winding, it is known as a compound machine.

(c) *The induction machine*

The polyphase induction motor operates with polyphase a.c. applied to the primary winding, usually on the stator, and carries a short-circuited secondary winding on the rotor. As its name suggests, the induction machine has e.m.f. induced in the short-circuited secondary winding by virtue of the primary rotating m.m.f. and, in many cases, no external electrical connection need to be made to the secondary circuit. Such a machine is then singly excited and is of the simplest form of construction with the winding on the rotor consisting of a series of copper or aluminium bars short-circuited together at each end and carried in a slotted magnetic structure. This so-called 'squirrel-cage' induction motor is probably the most widely used electromagnetic machine.

The mode of operation of the polyphase induction motor is illustrated in Fig. 1.8 and, once again, the conditions governing its operation are a particular case of the general machine. It is convenient to introduce the concept of fractional slip S given by:

$$S = \frac{\omega_s - \omega}{\omega_s} \tag{1.7}$$

at the earliest possible stage. The frequency f_2 Hz of the secondary currents is given by:

$$f_2 = \frac{\omega_2}{2\pi} \quad \text{so that} \quad f_2 = Sf \tag{1.8}$$

where f Hz is the supply frequency. At standstill, $\omega = 0$ so that $S = 1$ and $f_2 = f$. The machine then acts as a simple transformer with an air-gap and a short-circuited secondary winding. Thus a steady starting torque is produced and the polyphase induction motor is self-starting. At synchronous speed

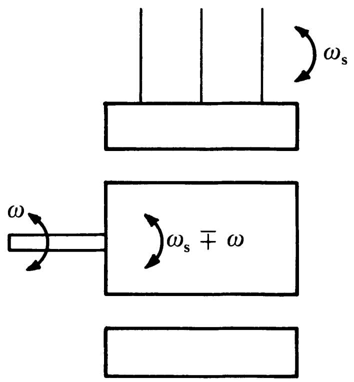


Fig. 1.8. The general machine as a 3-phase induction motor.

$\omega = \omega_s, S = 0$ and $f_2 = 0$. Since the rotor carries a polyphase short-circuited winding, a stationary m.m.f. cannot be produced and it must be concluded that at synchronous speed the value of the secondary m.m.f. will be zero. Thus no torque will be produced at synchronous speed and, since any practical machine must develop torque to overcome friction and windage losses, the practical polyphase induction motor cannot run at synchronous speed. The running-light or no-load speed will normally be of the order of 99·5% of synchronous speed corresponding to a slip of 0·005 and the full-load slip will be of the order of 0·05. Thus the polyphase induction motor is, effectively, a constant speed machine.

An induction machine can be made to generate if it is driven mechanically at above synchronous speed, so that $\omega > \omega_s$ and the slip is negative. If the machine is driven mechanically in the opposite direction to its primary rotating m.m.f. then the slip S becomes greater than unity and the machine acts as a brake. If, for example, the machine is operating normally as a loaded motor when two of the supply lines to the stator are reversed, the direction of the rotating m.m.f. will reverse. The rotor will then be rotating in the opposite direction to that of the rotating m.m.f., the machine will act as a brake and the speed will rapidly become zero. If, when zero speed has been reached, the supply is removed from the machine, this reversal of two supply lines is a useful method of rapidly stopping the motor and is generally known as plug-braking. If, however, the supply is not removed at zero speed, the machine will reverse its direction of rotation. The general idealized form of the torque–speed curve for a polyphase induction motor between rotor speed limits of $-\omega_s \leqslant \omega \leqslant 2\omega_s$, corresponding to a range of slips of $2 \leqslant S \leqslant -1$ is given in Fig. 1.9(a).

It is important to note at this stage that the previous discussion applies only to machines operating from a polyphase supply. In the particular case of the single-phase machine, the primary m.m.f. cannot be rotating and pulsates in phase with the variations in the single-phase primary current. Any pulsating m.m.f. can be resolved into two rotating m.m.f.s of equal magnitude, rotating in synchronism with the supply frequency but rotating in opposite directions. Then the slip, S_p, of the machine with respect to the m.m.f. rotating in the same direction as the rotor is given by the usual formula of (1.7) as:

$$S_p = \frac{\omega_s - \omega}{\omega_s} \tag{1.9}$$

and is known as the forward or positive-sequence slip. However, the slip, S_n, of the machine with respect to the m.m.f. rotating in the opposite direction to the rotor is given by:

$$S_n = \frac{-\omega_s - \omega}{\omega_s} = 2 - S_p \tag{1.10}$$

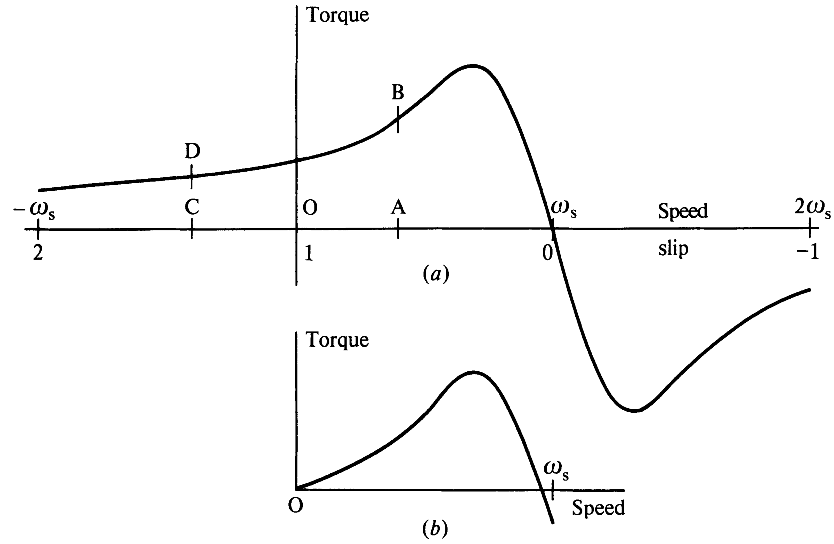


Fig. 1.9. ***Torque–speed curves for: (a) Polyphase induction motor. (b) Single-phase motor.***

and is known as the backward or negative-sequence slip. If these two components of m.m.f. are assumed to exist separately, the frequency and magnitude of the component e.m.f.s induced in the rotor by their presence will, in general, be different since $S_p \neq S_n$. Then the machine will, in general, produce a steady total torque as the algebraic sum of the component torques. At standstill, however, $\omega = 0$ and $S_p = S_n = 1$. Thus the component torques are equal and no starting torque is produced. It follows that the single-phase induction motor is not self-starting but that it will continue to rotate once started. Most such machines are, in practice, forms of asymmetrical 2-phase motors operated from a single-phase source. The form of the torque–speed curve for the single-phase motor can readily be obtained from that of the 3-phase machine. For any positive slip S = OA in Fig. 1.9(a), the positive-sequence torque is AB. Then, the negative slip $2 - S$ = OC such that OC = OA, and the negative-sequence torque is CD. The resultant torque is AB − CD and this process can be repeated for a range of values of slip ($1 \leqslant S \leqslant 0$) to give the form of the single-phase induction motor torque–speed curve of Fig. 1.9(b).

(d) The frequency changer

The general arrangement of the rotating frequency changer is given in Fig. 1.10 and the stator of such a machine carries no winding. The machine must be driven at speed ω and polyphase voltages are applied to the rotor slip-rings such that a rotor m.m.f., rotating at speed ω_s with respect to the rotor, is set up. Then, under the conditions governing the operation of the general machine, the polyphase voltage appearing at the commutator brushes will have an angular velocity of $\omega_s \pm \omega$ and frequency changing action takes place. The magnitude of the commutator voltage can easily be controlled by variation of the voltage applied to the slip-rings. If the phase angle of the commutator voltage relative to that at the slip-rings is to be controlled, then the position of the commutation brushes must be changed.

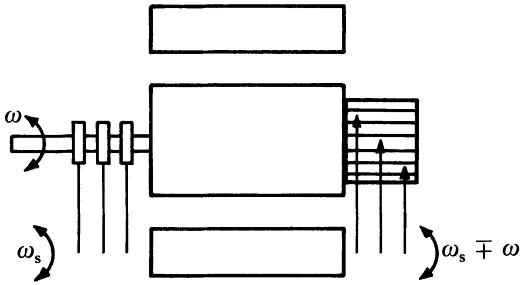


Fig. 1.10. The general machine as a 3-phase frequency changer.

It has previously been noted that the polyphase induction motor is, effectively, a constant speed machine. Speed control of such a machine can be obtained in various ways and, in the particular case of an induction motor with a rotor winding brought out through slip-rings, a method very popular in the past has used a rotating frequency changer. This method, known as the Leblanc system, is illustrated in Fig. 1.11 where the auxiliary drive for the frequency changer is the induction motor under control. Since both machines rotate at the same speed ω, it follows that the rotor circuit of the induction motor can be electrically connected to the commutator output in the manner shown in Fig. 1.11. If the magnitude of the voltage at the commutator is less than that at the motor slip-rings, energy will be transferred from the secondary of the induction motor, through the frequency changer and back into

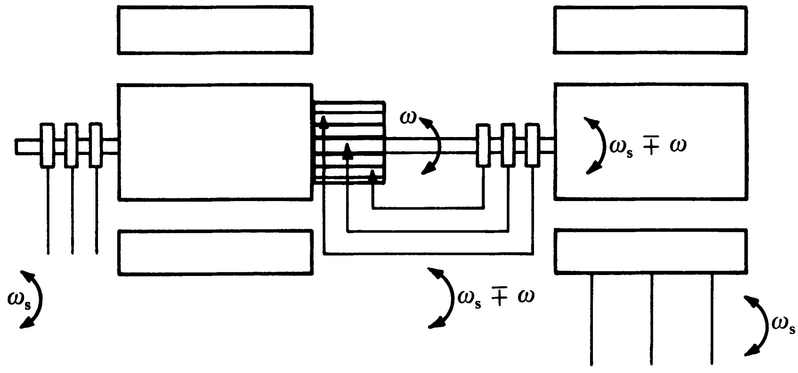


Fig. 1.11. Use of frequency changer for induction motor speed control.

the supply. Under these conditions, the motor will slow down and speed control below the synchronous speed can be obtained. When, however, the voltage at the commutator brushes exceeds that at the motor slip-rings, energy will be transferred from the supply, through the frequency changer and into the rotor circuit of the induction motor. This provides extra power for the induction motor and it can then be made to run above its synchronous speed. This method of control is a particular case of the use of injection of power into the rotor circuit of a slip-ring induction motor and the general method is capable of producing variation of the speed of the motor between wide limits, both above and below synchronous speed. It does, however, have two major disadvantages. Firstly, electrical connections must be made to the rotor circuit of the induction motor under control and the major advantage of this motor, that of no connection to the rotating parts, is lost. Secondly it involves the use of an auxliliary but expensive commutator machine. In many cases, however, the main motor and the commutator machine can be combined into one frame and two particular forms of polyphase a.c. commutator motor, the Schrage motor and the doubly fed motor, are still in relatively wide use.

1.5. Circuit conventions

A form of circuit convention to be used in any text should be established at an early stage. Various different conventions are in current use and the one used in this context is only one of many available. This circuit convention will be used continuously and consistently throughout the main body of this text.

The form of connection diagram representing a sink of electrical energy, i.e. an electrical motor, to be used throughout this text, is illustrated in Fig. 1.12, in which the armature current i in the direction specified will have a component in phase with the voltage rise e across the armature. The generated e.m.f. e is specified by Faraday's law in the general form:

$$e = \frac{d\lambda}{dt} \tag{1.11}$$

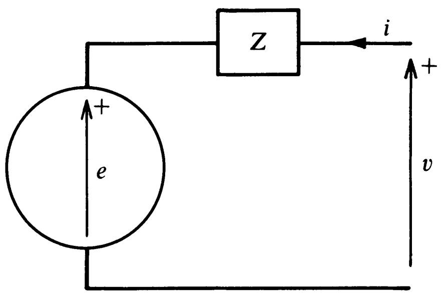


Fig. 1.12. Schematic representation of a sink of electrical energy.

where λ represents the flux linkages. The e.m.f. e acts in the direction shown so that, in general:

$$v = e + iz. \tag{1.12}$$

The torque T_e produced by the interaction of field and armature currents in the machine will result in the angular velocity ω and will be opposed by the load torque T_L on the shaft. Under steady-state conditions $T_e = T_L$, but under dynamic conditions when the speed is changing:

$$T_e = J\frac{d\omega}{dt} + T_L \tag{1.13}$$

where J is the polar moment of inertia of the rotating parts and $d\omega/dt$ is the angular acceleration.

In the case of a source of electrical energy, i.e. an electrical generator, electrical output energy will be obtained from mechanical input energy. The simplified form of connection representing a generator, to be used throughout this text, is given in Fig. 1.13, in which the armature current i in the direction specified will have a component in phase with the voltage rise e across the armature. The generated e.m.f. e is, again, specified by (1.11) such that, for a source:

$$v = e - iz. \tag{1.14}$$

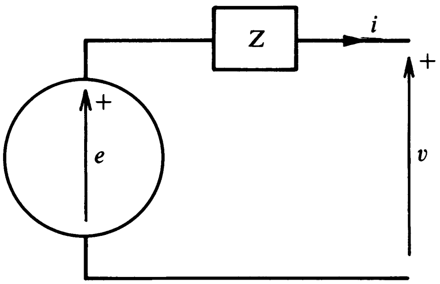


Fig. 1.13. Schematic representation of a source of electrical energy.

The torque T_s applied to the shaft of the generator will be opposed by the machine torque E_e and, under dynamic conditions:

$$T_s = J\frac{d\omega}{dt} + T_e. \tag{1.15}$$

In general most electromagnetic machines are reversible in that a motor can be made to generate or vice versa. However, mechanical drives and loads are generally irreversible and, in many cases, care must be taken to ensure that, should this electrical energy reversal occur, damage to the corresponding mechanical system does not take place.

Analysis of the performance of many electromagnetic machines involves the concept of mutually coupled circuits, and the so-called 'dot' convention to be used throughout this text to define the direction of induced voltage in a mutually coupled circuit is illustrated in Fig. 1.14. The voltage e_2 induced in circuit 2 of Fig. 1.14 by mutual coupling is given in magnitude by $e_2 = L_{12}$ $(\mathrm{d}i_1/\mathrm{d}t)$ and acts in the direction shown, i.e. a current 'entering' the dot in one winding will always induce a voltage in the second winding which is positive towards the dot in the second winding and vice versa. In many cases, the mutual inductance L_{12} is given the symbol M and in a reciprocal circuit, $L_{12} = L_{21} = M$.

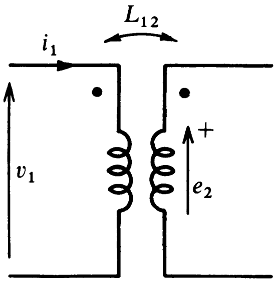


Fig. 1.14. The convention for mutually coupled circuits.

A practical form of a mutually coupled circuit is shown in Fig. 1.15 and is the equivalent of a sink (circuit 1) and a source (circuit 2). Then for circuit resistance R and self-inductance L with a mutual inductance L_{12}, the voltage–current relationships can be written in the form:

$$v_1 = \left(i_1 R_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t}\right) + e_1 \quad \text{(sink)} \tag{1.16}$$

$$v_2 = -\left(i_2 R + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t}\right) + e_2 \quad \text{(source)} \tag{1.17}$$

where $e_1 = -L_{12}\,\mathrm{d}i_2/\mathrm{d}t$ and $e_2 = L_{12}\,\mathrm{d}i_1/\mathrm{d}t$.

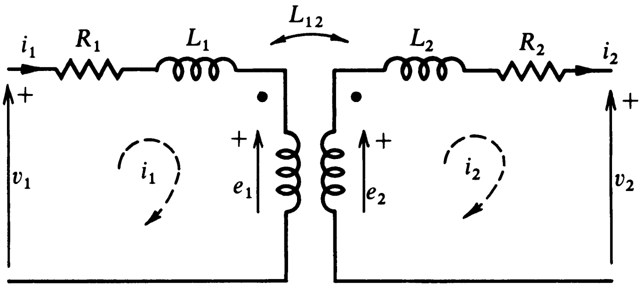


Fig. 1.15. A practical form of mutual coupling circuit.

In order to avoid confusion between instantaneous and steady (mean or root mean square) values, all instantaneous values will be in lower-case symbols (v, i, e, etc.) and steady values in capitals (V, I, E, etc.). There is an equal possibility of confusion between scalar and phasor quantities and all phasor quantities will be in bold type. Then, for the example $\mathbf{V} = \mathbf{I}(R + jX)$, the use of bold type signifies that **V** and **I** are phasor quantities having both magnitude and direction. The previous example can be rewritten in terms of the complex impedance $\mathbf{Z} = R + jX$, so that $\mathbf{V} = \mathbf{IZ}$. Each of the quantities **V**, **I** and **Z** can be specified in terms of magnitude and direction such that:

$$\mathbf{V} = V\angle\alpha, \qquad \mathbf{I} = I\angle\beta, \qquad \mathbf{Z} = Z\angle\theta = R + jX.$$

Then $V = IZ$ and $\alpha = \beta + \theta$.

It must be emphasized that the conventions listed are arbitrary. Other conventions can be used and the only requirement is that they be continuously and correctly applied throughout the text.

1.6 Normalization and per-unit values

In many cases, considerable simplification in calculations can be made by the use of the technique of normalization, such that the actual quantity is expressed as a fraction of some arbitrary value of that quantity.

One widely used particular form of normalization is the use of per-unit (p.u.) values. Most electrical equipment will be specified in terms of its rated volt-amperes S_N and its rated voltage V_N. The rated current I_N is then given by $I_N = S_N/V_N$ on a per-phase basis. Then, for an operating voltage **V**, current **I** and power P, per-unit voltage $\mathbf{V}_{pu} = \mathbf{V}/V_N$, per-unit current $\mathbf{I}_{pu} = \mathbf{I}/I_N$ and per-unit power $P_{pu} = P/S_N$.

Since equipment will not have a rated impedance the term base impedance Z_b must be introduced such that:

$$Z_b = \frac{V_N}{I_N} = \frac{V_N{}^2}{S_N}.$$

Then, for a circuit of impedance $\mathbf{Z} = R + jX$:

$$\mathbf{Z}_{pu} = \frac{\mathbf{Z}}{Z_b}, \qquad R_{pu} = \frac{R}{Z_b}, \qquad X_{pu} = \frac{X}{Z_b}.$$

The base admittance Y_b is given by $Y_b = 1/Z_b$, so that, for a circuit of admittance $\mathbf{Y} = G - jB$:

$$\mathbf{Y}_{pu} = \frac{\mathbf{Y}}{Y_b}, \qquad G_{pu} = \frac{G}{Y_b}, \qquad B_{pu} = \frac{B}{Y_b}.$$

In some cases, the impedance will be specified as a percentage value defined as the impedance voltage drop for rated current, expressed as a percentage of rated voltage. Then

$$\text{percentage impedance} = \frac{I_{\mathrm{N}}\mathbf{Z}}{V_{\mathrm{N}}} \times 100 = \mathbf{Z}_{\mathrm{pu}} \times 100$$

so that conversion from percentage to per-unit values can be made directly.

2. Armature winding arrangements

It has been noted in chapter 1 that machine windings can be classified as either field windings or armature windings. The forms taken by the field winding have been discussed in chapter 1, and the present chapter will be limited to forms of armature winding for both a.c. and d.c. machines. Such windings will generally be formed by wire wound coils of one or more turns placed in slots arranged to form either single-layer or double-layer windings.

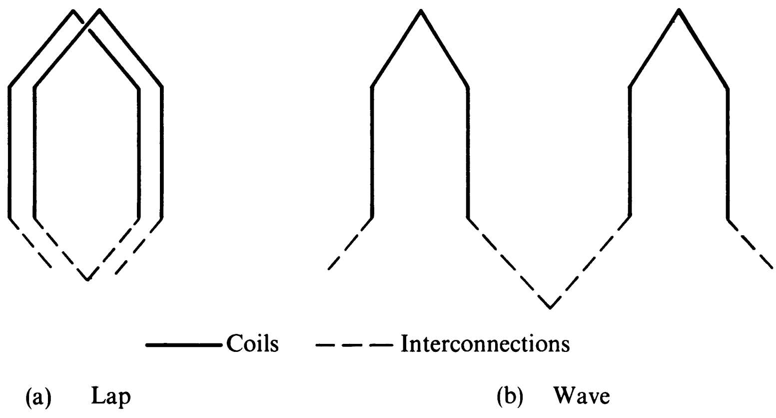


Fig. 2.1. Methods of interconnection of armature coils: (a) Lap connection. (b) Wave connection.

A single-layer winding is one in which one side of a coil occupies the whole of one slot whereas a double-layer winding is one in which there are two separate coil sides in any one slot. Most polyphase a.c. windings and armature windings for d.c. machines will be double-layer windings with multi-turn coils. A double-layer winding will, in general, have a lower leakage reactance and will produce a better waveform than the corresponding single-layer winding. The separate coils of a winding can be interconnected in several different manners and the two most general methods of interconnection, lap and wave connection, are illustrated in Fig. 2.1.

2.1. Polyphase winding arrangements

The armature winding of an a.c. machine can be considered in terms of several separate sections or 'phase belts'. A 3-phase double-layer winding can be produced with either three phase belts per pole pair (120° spread), or six phase belts per pole pair (60° spread). A 2-phase machine will normally have

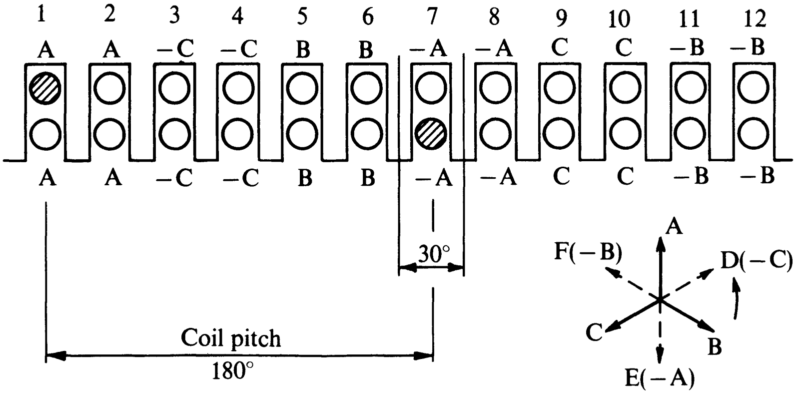


Fig. 2.2. Forms of polyphase windings with full-pitch coils:
(a) 3-phase, 60° spread winding.

a double-layer winding with four phase belts per pole pair (90° spread). The more usual arrangement is to use 60° spread for 3-phase windings and 90° spread for 2-phase windings, but it should be noted that a 3-phase, 60° spread winding is effectively a 6-phase winding and a 2-phase, 90° spread winding is effectively a 4-phase winding. A developed sectional diagram for a 12 slot,

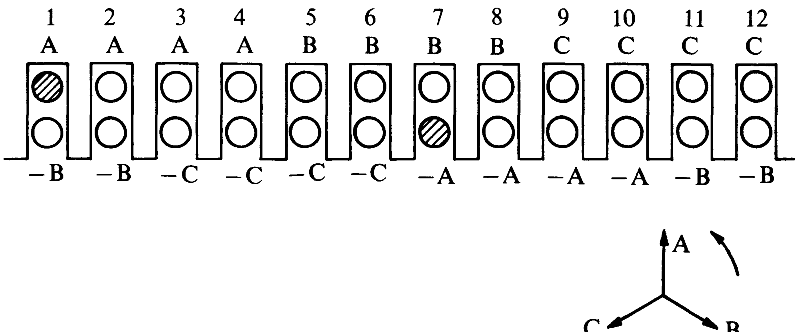


Fig. 2.2(b) 3-phase, 120° spread winding.

2-pole arrangement with a coil pitch of 6 slots (180° = slots 1–7) connected in each of the above forms is shown in Fig. 2.2 and it is apparent that the 3-phase, 60° spread form of connection illustrated in Fig. 2.2(a) will produce a sequence of ACBACB in the phase belts of the winding for a phase sequence

ABC. All the coils shown in Fig. 2.2 are full pitch coils in that the coil pitch is equal to the pole pitch. The pitch of a coil in a polyphase winding will often be less than the pole pitch and such an arrangement is known as a short-pitch or chorded winding. The effect of using chorded coils is to reduce the length

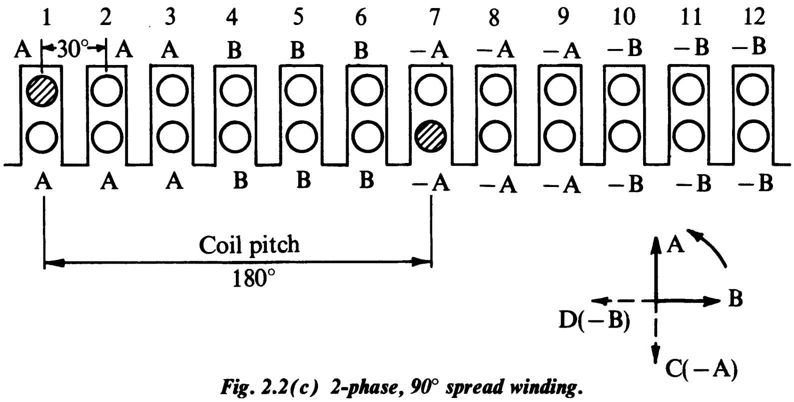


Fig. 2.2(c) 2-phase, 90° spread winding.

of the end connection and to reduce the magnitude of certain harmonics in the waveform of the generated e.m.f. It is standard practice to use chorded windings in most a.c. machines and the coil pitch will generally be between 2/3 pole pitch and full pole pitch. The developed diagram corresponding to

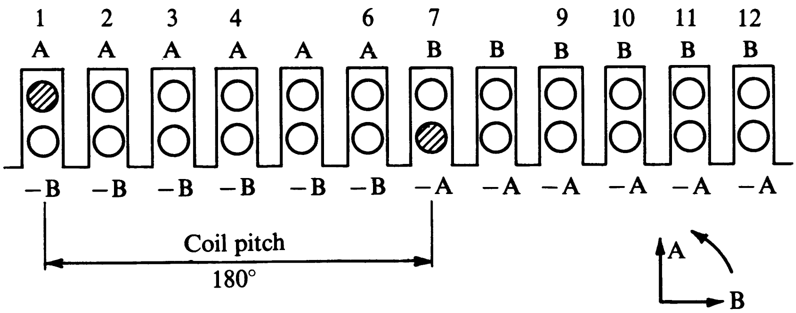


Fig. 2.2(d) 2-phase, 180° spread winding.

a 12 slot, 2-pole, 3-phase 60° spread arrangement with a coil pitch of 5/6 pole pitch (150°e = slots 1–6) is illustrated in Fig. 2.3.

Simplified 'clock' diagrams illustrating the arrangement of the phase belts for one layer of a 3-phase, double-layer 60° spread, 2-pole, 4-pole, and 6-pole winding are shown in Fig. 2.4, and it can be seen that for a phase-sequence of ABC, the origin sequence of the points X, Y, and Z displaced by 120°m in space is ABC for the 2-pole case, ACB for the 4-pole case, and AAA for the 6-pole case. It immediately follows that if entry points displaced by 120°m are made to the phases of a 3-phase machine with a double-layer winding with $6K + 2$ poles, where K is an integer (i.e., 2, 8, 14, 20, poles) they must be made in the order of the phase sequence, ABC, and that for a machine with $6K + 4$ poles (i.e., 4, 10, 16, 22, poles), they must be made in the order of reversed phase sequence, ACB. For a machine with $6K$ poles (i.e., 6, 12, 18,

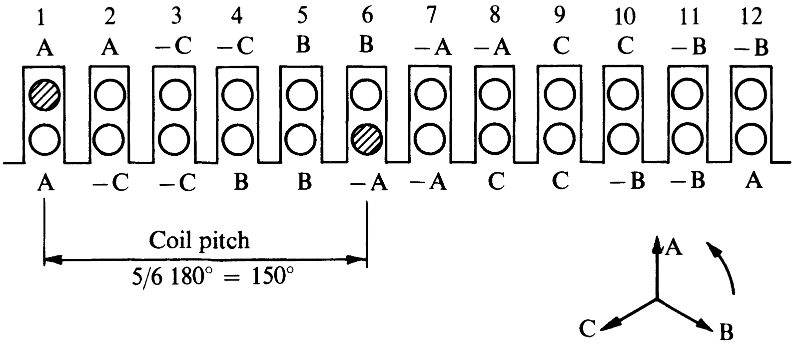


Fig. 2.3. Form of a 3-phase, 60° spread winding with 5/6 pitch coils.

poles) it is never possible to make entry points displaced by 120°m. In many practical cases, the points of entry to the three phase windings will be as close together as possible and are shown as the arrowed lines in Fig. 2.4. In all cases, the arrangement of the other layer of the double-layer winding will be identical to that shown in Fig. 2.4, displaced by an angle corresponding to the

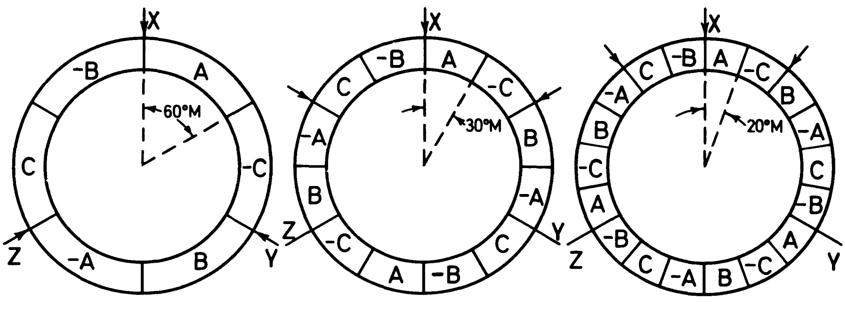


Fig. 2.4. Clock diagrams for a 3-phase, 60° spread winding:
(a) 2-pole arrangement.
(b) 4-pole arrangement.
(c) 6-pole arrangement.

coil pitch and the arrangement of both layers of the winding for the 4-pole case, shown in Fig. 2.4, when the coil pitch is 5/6 pole pitch (75°m) is shown in Fig. 2.5.

Consider the particular case of a 3-phase machine with a 36 slot stator having a double-layer winding with a coil pitch of 90°m (9 slots). If this winding is to be arranged in a 3-phase, 4-pole, lap connected form, there will be 36/(3 × 4) = 3 coils/pole/phase (c.p.p. = q). Such an arrangement for which the c.p.p. is an integer is known as an integral slot winding. Thus each phase belt of 60°e will occupy 3 slots of the stator. The phase and polarity of

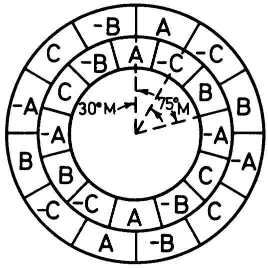


Fig. 2.5. Clock diagram for both layers of a 3-phase, 60° spread, double layer, 4-pole winding with 5/6 pitch coils.

all 72 coil sides forming this winding is illustrated in Fig. 2.6, in which it is assumed that the first phase, A, belt occupies slots 1, 2, and 3. It is important to note that, when the coil side in the bottom of slot 1 is assumed to have a 'positive' polarity it follows that the coil side in the top of slot 10 must have a 'negative' polarity, since a coil is formed between these two coil sides. This process must be repeated for corresponding coil sides in all other slots. It is apparent that the four phase belts of phase A must be connected in series,

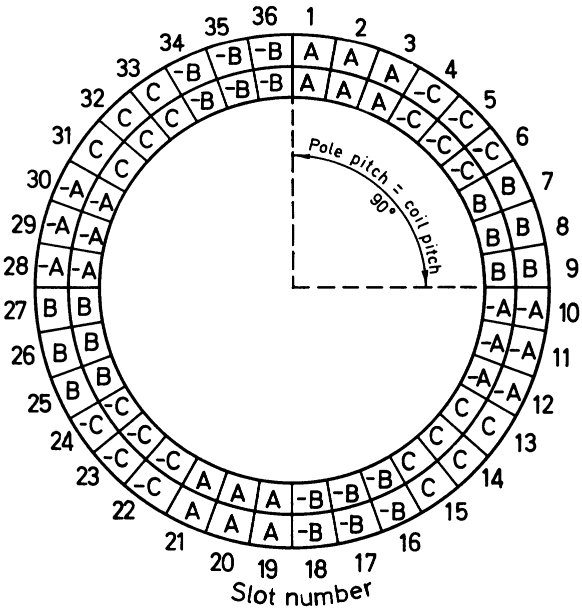


Fig. 2.6. Phase and polarity of coil sides for a 3-phase, 60° spread, double-layer, 4-pole winding in 36 slots.

parallel, or series/parallel to form the phase A winding and the method of interconnection will depend on the operating voltage of the machine. It is, however, standard practice to connect pairs of phase belts in any one phase in series opposition, so that the three coils formed between the bottoms of slots 1, 2, and 3, and the tops of slots 10, 11, and 12, respectively, in Fig. 2.6, will be connected in series opposition with the three coils formed between the bottoms of slots 10, 11, and 12, and the tops of slots 19, 20, and 21 respectively to form two series connected phase belts. The other two coil groups in phase A will be connected in an identical manner and it is then possible to connect the two separate groups of phase belts in phase A in either series or parallel.

2.2. D.c. machine armature windings

These windings are always of the closed type of double-layer lap or wave form in the forms illustrated in Fig. 2.1 connected to a commutator. The simplex lap winding will have as many parallel paths as there are poles, while for the simplex wave winding, there will always be two parallel paths. A multiplex winding is specified by the number of parallel paths in the winding relative to that for the corresponding simplex winding.

The arrangement of the interconnections between coils and the method of connection to the commutator can be specified in terms of the back pitch y_b, and the front pitch y_f of the coil and the commutator pitch y_c. The back pitch of any d.c. armature winding will generally be as nearly as possible equal to the pole pitch, so that for a P-pole winding in Q slots the back pitch will be the integer equal to or less than Q/P. In all cases, the commutator pitch will equal the algebraic sum of the front pitch and the back pitch of the coil.

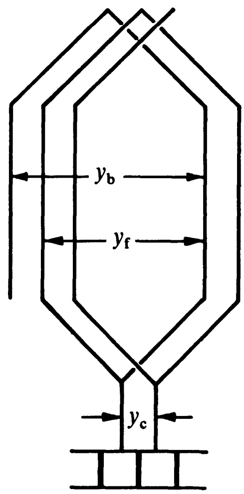


Fig. 2.7. Simplified form of lap connected winding for a d.c. machine.

The commutator pitch for a simplex lap winding will always be one commutator segment and the commutator pitch for a multiplex lap winding of degree *m* will be *m* commutator segments. Simplified arrangements of a lap winding are illustrated in Fig. 2.7.

The commutator pitch y_c for a simplex wave winding is governed by the equation

$$y_c = \frac{S \pm 1}{\text{pole pairs}} \tag{2.1}$$

where S is the number of commutator segments and a simplex wave winding can only be obtained if y_c is an integer. In some cases where y_c is not an integer, a so-called 'dummy coil' is introduced into the winding. This coil will normally be on open circuit and its function will be to maintain mechanical symmetry of the armature. Simplified arrangements of a wave winding are illustrated in Fig. 2.8.

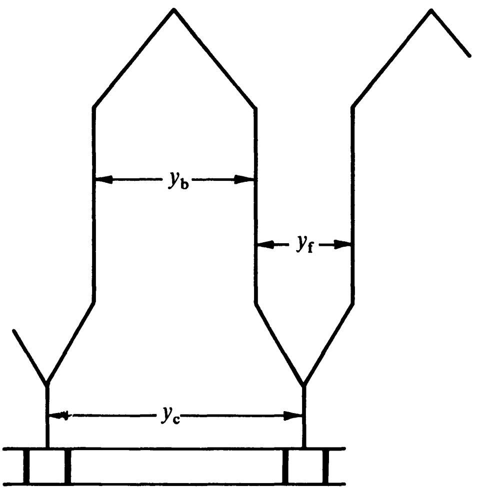


Fig. 2.8. Simplified form of wave connected winding for a d.c. machine.

Consider a 20-slot armature carrying a 4-pole, double-layer, simplex lap connected winding. The pole pitch is 90°m and the back pitch of the coil will be the integer less than or equal to 20/4, i.e., 5 slots. The commutator pitch will be chosen as $+1$ so that the winding is progressive, and it follows that the front pitch of the coil is given by $y_f = -4$. The winding diagram in radial form is shown in Fig. 2.9 and it should be noted that there are four parallel paths through the armature. The brushes on the commutator will be situated as nearly as possible on the centre of the poles so that the coil undergoing commutation is situated in the interpolar gap. As a particular armature coil rotates, it must pass from the influence of one pole (say, a north pole) to the influence of an adjacent pole (a south pole), and during this process there must be a reversal of current through the coil with an associated short-circuited position. It follows that this reversal of current should take place when the coil is situated in the interpolar gap.

It has so far been assumed that the machine is geometrically and magnetically similar so that the generated voltages in each of the parallel paths of a lap connected winding will be equal. In practice, these voltages will not have

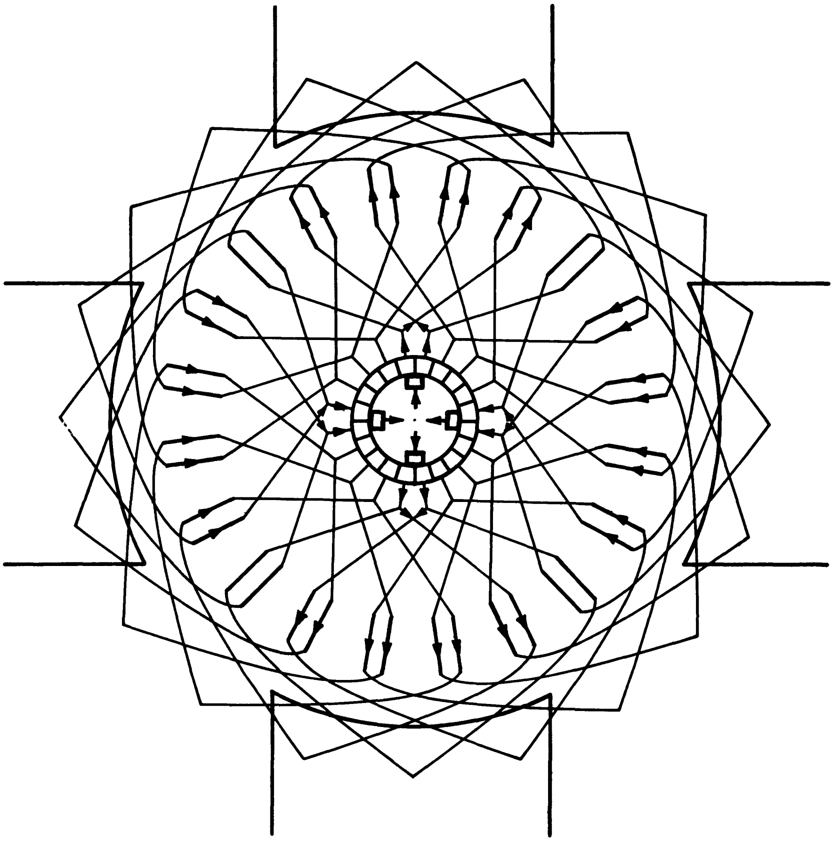


Fig. 2.9. A double-layer, simplex lap connected, 4-pole d.c. armature winding.

exactly the same value and circulating currents through the armature and brushes will exist. In these circumstances, most lap connected machines will be fitted with 'equalizers', in the form of copper straps with a large cross-sectional area, which join together points on the winding exactly 360°e apart. This process of equalizing requires that the number of slots per pole pair is an integer and the major advantage of equalization is that excessive brush currents with associated commutation difficulties can be avoided. A lap connected winding is said to be 100 per cent equalized when all points on the winding exactly 360°e apart are joined together and, for a simplex winding, the number of equalizer bars is given by the slots per pole pair. For the case shown in Fig. 2.9, ten equalizer connections would be required, each of which would join two coils together.

As an example of a simplex wave connected winding, consider a 17-slot armature with a double-layer winding connected for 4-pole operation. The commutator pitch y_c is given by $(17 \pm 1)/2 = 8$ or 9 and will be taken as 9. The back pitch of the coil will be the integer less than 17/4, i.e., 4 slots so that the front pitch of the coil will be 5 slots. The complete arrangement of this winding in radial form is shown in Fig. 2.10.

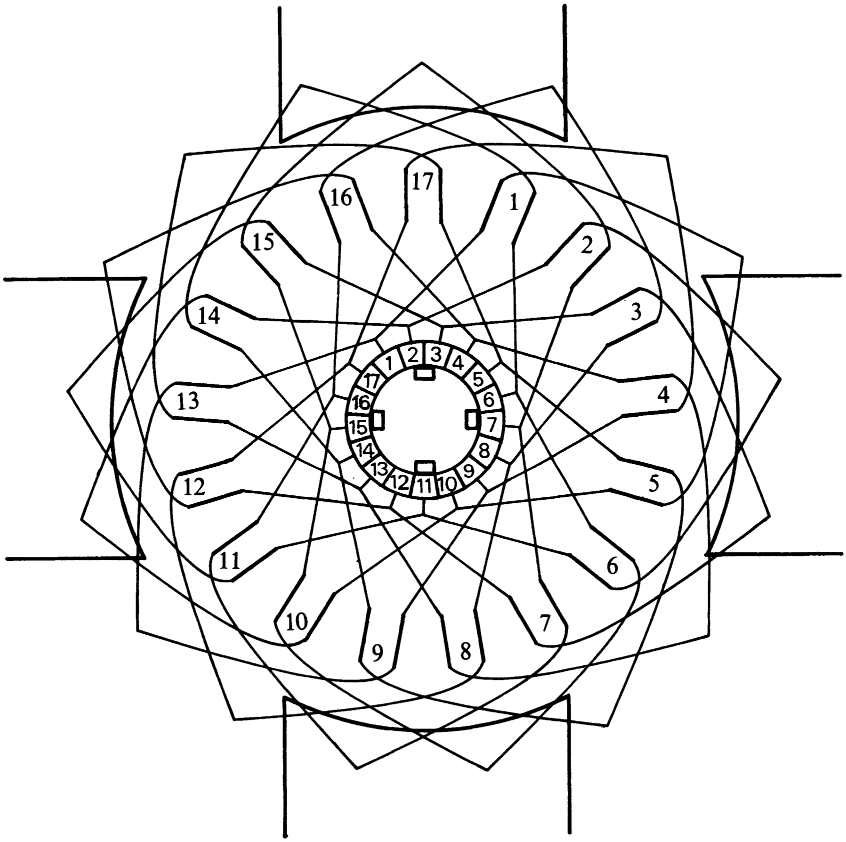


Fig. 2.10. A double-layer, simplex wave connected, 4-pole d.c. armature winding.

In many cases, there will be more than two coil sides in any one slot so that the number of commutator segments will be greater than the number of armature slots. This has the major advantage that, for a given output voltage, the voltage between adjacent segments decreases as the number of segments increases with a consequent reduced risk of flashover between segments.

The main difference between simplex lap and wave connected armature winding is the number of parallel paths through the armature. The voltage developed across each of the parallel paths in a lap connected winding must equal the system voltage, although the current in each of the parallel paths will be considerably less than the total armature current. The wave connected winding has only two parallel paths through the armature so that the current in each path will be one-half of the total armature current. In practice, the wave connected winding would generally be used in cases where the total armature current was less than 500 A and the lap connected winding fitted with the necessary equalizers, in applications where armature currents greater than 500 A are necessary.

Tutorial Problems

1. A 72-slot stator is to be wound for 3-phase operation. If the coil pitch is 6 slots, derive a suitable form for the arrangement as (a) an 8-pole winding, (b) a 10-pole winding and draw a clock diagram to show the position of the phase belts in each layer of the winding in each case.

2. A 36-slot stator carries a double-layer winding with a coil pitch of 13 slots and is to be wound with a 3-phase, 8-pole winding. Derive a suitable winding arrangement and find its fundamental pitch and distribution factors.

   (*Answer:* 0·985; 0·975)

3. A 48-slot stator carries two identical separate single-layer windings, each with a coil pitch of 1–12 = 11 slots. Show how such windings can be connected to give the m.m.f. distribution associated with a 4-pole, 3-phase, double-layer, 60° spread winding in 48 slots with (a) full pitch coils, (b) 5/6 pitch coils.

4. Draw a developed diagram for a 6-pole, double-layer d.c. armature winding in 28 slots for lap connection. Draw the brushes and the necessary paralleling connections. The winding should be progressive.

3. Process of energy conversion

The conditions governing the generation of e.m.f. and the production of torque in a heteropolar electrical machine, in which the coupling field is electromagnetic, require that there shall be a time rate of change of flux linkage between the armature conductors and the field flux.

In the case of a d.c. machine, the flux is stationary and the coil moving; for a normal synchronous machine the flux is rotating and the coil stationary, while for the polyphase induction motor, the coil moves with respect to a constant rotating flux. For the ideal case, the field set up in a d.c. machine will be constant over the pole, while that of an a.c. machine will be sinusoidally distributed in space. In practice, however, the presence of space harmonics in the field flux wave and the armature m.m.f. wave of an a.c. machine can be of considerable importance but no account of harmonic effects will be taken in this simple treatment.

3.1. Electromechanical energy conversion

The process of electromechanical energy conversion must always satisfy the conservation of energy, so that in the case of a sink of electrical energy, i.e., a motor, it follows that:

$$\text{Electrical input energy} = \text{Mechanical output energy} + \text{Energy loss} + \text{Increase in stored energy.} \quad (3.1)$$

The energy losses associated with this form of energy conversion are the energy loss because of the resistances of the circuits, the energy loss associated with the coupling field and the friction and windage loss always associated with motion. In most cases, the coupling field will be a magnetic field with its associated small iron losses and this text will be limited to the treatment of devices of this form. Then the nett electrical input W_e can be equated to the increase in energy stored in the magnetic field W_f and the mechanical output energy W_m so that, in incremental form:

$$\mathrm{d}W_e = \mathrm{d}W_f + \mathrm{d}W_m. \quad (3.2)$$

The incremental mechanical energy for a virtual displacement dx in time dt

when the force is f can be written:

$$dW_m = f\,dx \tag{3.3}$$

and it follows from (3.2) and (3.3) that:

$$f = \frac{\partial W_e}{\partial x} - \frac{\partial W_f}{\partial x}. \tag{3.4}$$

The induced e.m.f. e can be written, by Faraday's law, as:

$$e = \frac{d\lambda}{dt} \tag{3.5}$$

where λ represents the flux linkages. Then the nett incremental electrical energy in time dt can be written:

$$dW_e = ei\,dt = \left(\frac{d\lambda}{dt}\right)i\,dt = i\,d\lambda.$$

Then

$$f = \frac{\partial(i\,d\lambda)}{\partial x} - \frac{\partial W_f}{\partial x}. \tag{3.6}$$

If the displacement occurs with the flux linkages λ held constant it follows from (3.6) that:

$$f = -\left(\frac{\partial W_f}{\partial x}\right)_{\lambda\,\mathrm{const}}. \tag{3.7}$$

The general form of the relationship between the flux linkages λ and the current i is that of the normal magnetization curve shown in Fig. 3.1 for

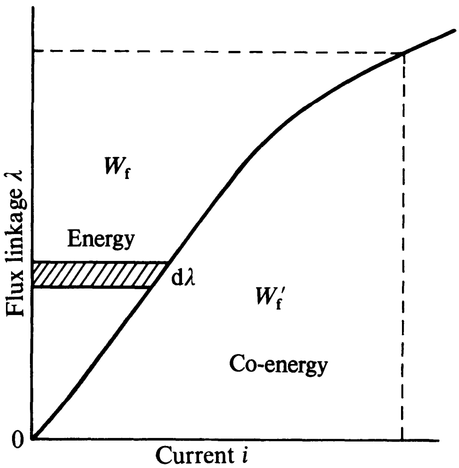


Fig. 3.1. The magnetization curve.

which the energy stored in the magnetic field is the area shown and is given by:

$$W_f = \int_0^\lambda i\,d\lambda. \tag{3.8}$$

The area between the x-axis and the curve must also have the dimensions of energy and is known as the co-energy W_f' such that:

$$W_f' = \int_0^i \lambda\,di \tag{3.9}$$

and
$$W_f + W_f' = \lambda i,$$

so that
$$W_f = \lambda i - W_f'.$$

Then, from (3.2) and (3.3)

$$dW_e = d(\lambda i - W_f') + f\,dx.$$

Now
$$dW_e = i\,d\lambda$$

so that
$$f = -\lambda\frac{\partial i}{\partial x} + \frac{\partial W_f'}{\partial x}. \tag{3.10}$$

If the displacement occurs at constant current it follows from (3.10) that

$$f = \left(\frac{\partial W_f'}{\partial x}\right)_{i\,\text{const}}. \tag{3.11}$$

The foregoing analysis has been concerned with force f and linear displacement x. In the case of a rotational transducer, torque T and angular displacement θ must be introduced and (3.7) and (3.11) can be written:

$$T = -\left(\frac{\partial W_f}{\partial \theta}\right)_{\lambda\,\text{const}} = \left(\frac{\partial W_f'}{\partial \theta}\right)_{i\,\text{const}}. \tag{3.12}$$

For the particular case of a linear magnetic circuit, the stored energy is equal to the co-energy and for a singly excited system is given by:

$$W_f = W_f' = \tfrac{1}{2}\lambda i = \tfrac{1}{2}F\Phi \tag{3.13}$$

where $F = Ni$ is the m.m.f. and Φ is the flux.

The reluctance S of the magnetic circuit is given by $F = S\Phi$ so that:

$$W_f = W_f' = \tfrac{1}{2}\Phi^2 S. \tag{3.14}$$

The inductance L is given by $\lambda = Li$ so that:

$$W_f' = \tfrac{1}{2}Li^2.$$

Then from (3.7)

$$f = -\left(\frac{\partial W_f}{\partial x}\right)_{\lambda\,\text{const}} = -\tfrac{1}{2}\Phi^2 \frac{dS}{dx} \tag{3.15}$$

and from (3.11)

$$f = \left(\frac{\partial W_f'}{\partial x}\right)_{i\,\text{const}} = \tfrac{1}{2}i^2 \frac{dL}{dx}. \tag{3.16}$$

For the case of a rotational transducer (3.15) and (3.16) become:

$$T = -\tfrac{1}{2}\Phi^2 \frac{dS}{d\theta} \tag{3.17}$$

$$T = \tfrac{1}{2}i^2 \frac{dL}{d\theta}. \tag{3.18}$$

In the case of the simple doubly excited system shown in Fig. 3.2, the total flux linkage λ_1 with circuit 1 will be given by:

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

where L_{11} is the self-inductance of circuit 1 and L_{12} is the mutual inductance between the two circuits.

In a similar manner, the total flux linkage λ_2 with circuit 2 is given by:

$$\lambda_2 = L_{12}i_1 + L_{22}i_2$$

where L_{22} is the self-inductance of circuit 2.

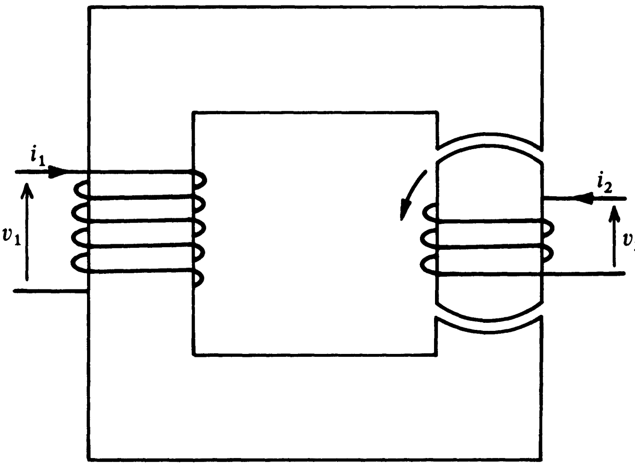


Fig. 3.2. A doubly excited electromechanical transducer.

The total energy W_f stored in the coupling field is given by:

$$W_f = \tfrac{1}{2}L_{11}i_1^2 + \tfrac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2.$$

The torque T can be written in the form:

$$T = \left(\frac{\partial W_f}{\partial \theta}\right)_{i\,\text{const}} = \tfrac{1}{2}i_1^2\frac{\mathrm{d}L_{11}}{\mathrm{d}\theta} + \tfrac{1}{2}i_2^2\frac{\mathrm{d}L_{22}}{\mathrm{d}\theta} + i_1i_2\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta}. \tag{3.19}$$

The voltage–current relationships for the two circuits can be written as:

$$v_1 = i_1r_1 + \frac{\mathrm{d}\lambda_1}{\mathrm{d}t}$$

and

$$v_2 = i_2r_2 + \frac{\mathrm{d}\lambda_2}{\mathrm{d}t}. \tag{3.20}$$

In general, the inductances L_{11}, L_{22}, and L_{12} will be functions of the angular position θ of the rotor and the currents will be time functions so that:

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}[L_{11}(\theta)i_1(t) + L_{12}(\theta)i_2(t)]$$

or

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = L_{11}\frac{\mathrm{d}i_1}{\mathrm{d}t} + i_1\frac{\mathrm{d}L_{11}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + L_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t} + i_2\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t}. \tag{3.21}$$

In a similar manner:

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = L_{12}\frac{\mathrm{d}i_1}{\mathrm{d}t} + i_1\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + L_{22}\frac{\mathrm{d}i_2}{\mathrm{d}t} + i_2\frac{\mathrm{d}L_{22}}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t}. \tag{3.22}$$

When the values of $\mathrm{d}\lambda_1/\mathrm{d}t$ and $\mathrm{d}\lambda_2/\mathrm{d}t$ from (3.21) and (3.22) are substituted in (3.20) it follows that:

$$v_1 = \left[i_1r_1 + L_{11}\frac{\mathrm{d}i_1}{\mathrm{d}t}\right] + \left[\left(i_1\frac{\mathrm{d}L_{11}}{\mathrm{d}\theta} + i_2\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta}\right)\frac{\mathrm{d}\theta}{\mathrm{d}t}\right] + \left[L_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right] \tag{3.23}$$

and

$$v_2 = \left[i_2r_2 + L_{22}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right] + \left[\left(i_1\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta} + i_2\frac{\mathrm{d}L_{22}}{\mathrm{d}\theta}\right)\frac{\mathrm{d}\theta}{\mathrm{d}t}\right] + \left[L_{12}\frac{\mathrm{d}i_1}{\mathrm{d}t}\right]. \tag{3.24}$$

The first terms on the right-hand side of (3.23) and (3.24) represent the so-called 'voltage of self-impedance', the second terms represent the so-called 'speed voltage', and the third terms represent the so-called 'transformer voltage'.

In many cases, the self-inductances L_{11} and L_{22} will not depend on the angular position of the rotor so that (3.19), (3.23), and (3.24) reduce to:

$$T = i_1i_2\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta} \tag{3.25}$$

$$v_1 = \left(i_1 r_1 + L_{11}\frac{\mathrm{d}i_1}{\mathrm{d}t}\right) + \left(i_2\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta} + L_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t} \tag{3.26}$$

$$v_2 = \left(i_2 r_2 + L_{22}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right) + \left(i_1\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\frac{\mathrm{d}L_{12}}{\mathrm{d}\theta} + L_{12}\frac{\mathrm{d}i_1}{\mathrm{d}t}. \tag{3.27}$$

EXAMPLE 3.1. Many single-phase synchronous machines will be made in the form illustrated in Fig. 3.3, and such a machine is known as a single-phase

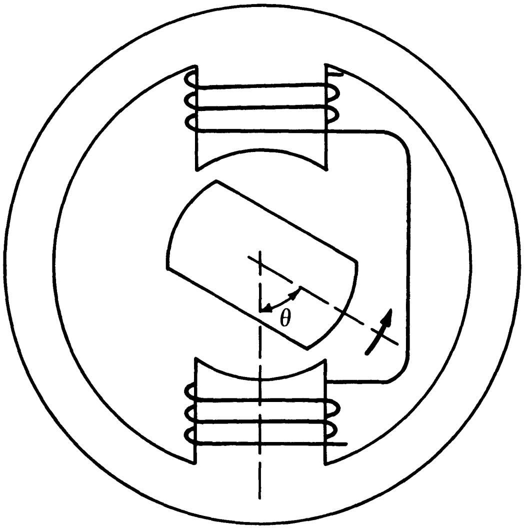


Fig. 3.3. The single-phase reluctance motor.

reluctance motor. The air-gap of the machine is not constant and the reluctance S will be assumed to vary periodically with the angular position θ of the rotor between a value S_q when $\theta = 90°$ or $270°$ and a value S_d when $\theta = 0°$ or $180°$. The reluctance can be written as:

$$S = \frac{S_q + S_d}{2} - \frac{S_q - S_d}{2}\cos 2\theta.$$

The torque T is given from (3.17) as

$$T = -\tfrac{1}{2}\Phi^2\frac{\mathrm{d}s}{\mathrm{d}\theta} = -\tfrac{1}{2}\Phi^2(S_q - S_d)\sin 2\theta.$$

If the flux Φ is assumed to be given by $\Phi = \Phi_m \cos \omega_s t$, it follows that

$$T = -\tfrac{1}{2}\Phi_m{}^2(\cos^2 \omega_s t)(S_q - S_d)\sin 2\theta.$$

But $$\cos^2 \omega_s t = \tfrac{1}{2}(1 + \cos 2\omega_s t)$$

so that $T = -\tfrac{1}{4}\Phi_m^2(S_q - S_d)(\sin 2\theta + \sin 2\theta \cos 2\omega_s t)$

or $T = -\tfrac{1}{4}\Phi_m^2(S_q - S_d)\left[\sin 2\theta + \tfrac{1}{2}\sin 2(\theta + \omega_s t) + \tfrac{1}{2}\sin 2(\theta - \omega_s t)\right].$

If $\theta = \omega t - \delta$, such that δ is the displacement of the rotor when $t = 0$, the torque equation can be written:

$$T = \frac{-\Phi_m^2}{4}(S_q - S_d)\left[\sin 2(\omega t - \delta) + \tfrac{1}{2}\sin(2(\omega + \omega_s)t + 2\delta) + \tfrac{1}{2}\sin(2(\omega - \omega_s)t - 2\delta)\right].$$

The average value T_A of this torque will be zero unless $\omega = \omega_s$, and then:

$$T_A = \frac{\Phi_m^2}{8}(S_q - S_d)\sin 2\delta. \tag{3.28}$$

Thus the reluctance motor will only produce steady torque at synchronous speed and this therefore is a synchronous machine. The angle δ is the load angle of the machine and the torque (or power)–load angle characteristic of the machine will be sinusoidal with maximum torque at $\delta = 45°$. Such a torque is known as reluctance torque and is produced by the geometric asymmetry of the rotor.

EXAMPLE 3.2. The dynamometer type of instrument illustrated in Fig. 3.4 consists of a fixed coil 1, and a moving coil 2, suspended on a pivot acting against a spring. The self-inductances of the two coils are independent of the angular displacement θ between them and the mutual inductance L_{12} between the coils will be a maximum when $\theta = 0°$ and zero when $\theta = 90°$ so that:

$$L_{12} = M\cos\theta.$$

In the linear case the torque is given by:

$$T = \left(\frac{\partial W_f'}{\partial\theta}\right)_{i\,\text{const}} = \left(\frac{\partial W_f}{\partial\theta}\right)_{\lambda\,\text{const}}$$

But $W_f = \tfrac{1}{2}L_{11}i_1^2 + \tfrac{1}{2}L_{22}i_1^2 + i_1 i_2 M\cos\theta$

then $T = -i_1 i_2 M\sin\theta.$

When the instrument is used as an a.c. ammeter

$$i_1 = i_2 = I_m\cos\omega_s t$$

so that

$$T = -I_m^2 M\sin\theta\cos^2\omega_s t = \frac{-I_m^2 M\sin\theta}{2}(1 + \cos 2\omega_s t).$$

The average value T_a of this torque per cycle of the alternating current will be given by:

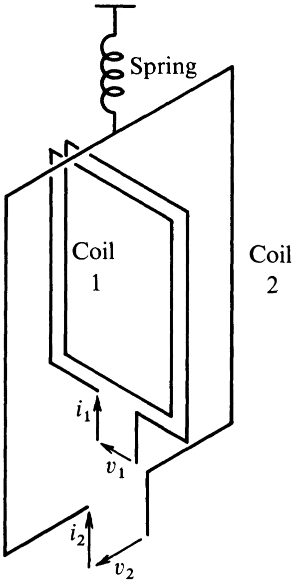


Fig. 3.4. The dynamometer instrument.

$$T_a = \frac{-I_m{}^2 M \sin\theta}{2}.$$

Thus the deflection of the moving coil will be proportional to $(\text{current})^2$ for small displacements.

When the instrument is used as an a.c. wattmeter, the system voltage $V_1 = V_m \cos \omega_s t$ will be applied to the moving coil of resistance R_2 and negligible inductance and the system current $i = I_m \cos(\omega_s t - \phi)$ will be applied to the fixed coil.

Then $$i_1 = I_m \cos(\omega_s t - \phi), \qquad i_2 = \frac{V_m}{R_2}\cos \omega_s t.$$

Thus $$T = \frac{V_m I_m}{R}(\cos \omega_s t)\cos(\omega_s t - \phi) M \sin\theta$$

$$= \frac{-V_m I_m}{2R} M \sin\theta \left[\cos\phi - \cos(2\omega_s t - \phi)\right].$$

The average value T_w of the torque is given by:

$$T_w = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi \frac{M}{R}\sin\theta$$

so that the deflection is proportional to power.

3.2. Generation of e.m.f.

In the basic analysis, the winding will be assumed to be concentrated with full-pitch coils.

Consider the case of a sinusoidally distributed flux, such that the flux density in the air-gap varies sinusoidally around its periphery. When an armature coil of N turns is displaced by an angle $\frac{1}{2}p\theta$ from the position of maximum flux density, the flux linkage λ with the coil as a function of time is given by

$$\lambda = N\Phi_m \sin \omega_s t \tag{3.29}$$

where Φ_m is the maximum value of flux and $\omega_s t = \frac{1}{2}p\theta$. Then

$$e = \frac{d\lambda}{dt} = N\Phi_m \omega_s \cos \omega_s t + N \sin \omega_s t \frac{d\Phi_m}{dt} \tag{3.30}$$

where the frequency, f hertz, of the generated e.m.f. is given by

$$2\pi f = \omega_s = \frac{d}{dt}\left(\frac{p\theta}{2}\right). \tag{3.31}$$

The first term in (3.20) is known as a speed voltage and the term involving $d\Phi_m/dt$, which is only present when the maximum value of flux density is a function of time, is known as a transformer voltage.

Then the speed voltage in (3.30) can be written

$$e = 2\pi f N\Phi_m \cos 2\pi f t \tag{3.32}$$

and its r.m.s. value E is given by

$$E = \frac{2\pi f N\Phi_m}{\sqrt{2}} = 4{\cdot}44 f N\Phi_m. \tag{3.33}$$

For the case of a d.c. machine with a flux per pole Φ, the total flux cut by one conductor per revolution will be Φp. If the speed of rotation is N r.p.m. the e.m.f. generated in a single conductor will be given by

$$e = \frac{\Phi p N}{60}.$$

For an armature with z conductors and a parallel paths, the total generated e.m.f. E is given by

$$E = \frac{p}{a}\frac{\Phi N z}{60}. \tag{3.34}$$

The angular velocity ω rad/s is given by $\omega = 2\pi N/60$ so that

$$E = \frac{pz}{2\pi a}\Phi\omega \tag{3.35}$$

or

$$E = K\Phi\omega. \tag{3.36}$$

3.3. Winding factors for a.c. machine windings

In practice, the coils of an armature winding will not be concentrated but distributed in space so that the induced voltages in each coil will not be in phase but will be displaced from each other by the slot angle α_s. The effect of coil distribution is taken into account by introducing the distribution factor k_d defined as the ratio of the actual resultant e.m.f. E_r to the arithmetic sum of the individual coil e.m.f. E. For a winding with q coils/pole/phase, the distribution factor is given by

$$k_d = \frac{E_r}{qE}. \tag{3.37}$$

The arrangement for the particular case of $q = 3$ is illustrated in Fig. 3.5, and it follows from Fig. 3.5 that

$$E_r = 2X \sin \frac{q\alpha_s}{2} \quad \text{and} \quad E = 2X \sin \frac{\alpha_s}{2}.$$

Then

$$k_d = \frac{\sin \dfrac{q\alpha_s}{2}}{q \sin \dfrac{\alpha_s}{2}}. \tag{3.38}$$

where α_s is measured in electrical units. When the winding is a 3-phase, 60° spread double-layer winding it follows that $q\alpha_s = 60°\text{e} = \frac{1}{3}\pi$. Then

$$k_d = \frac{\sin \dfrac{\pi}{6}}{q \sin \dfrac{\pi}{6q}} = \frac{0{\cdot}5}{q \sin \dfrac{\pi}{6q}}. \tag{3.39}$$

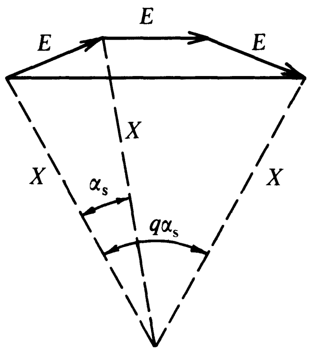


Fig. 3.5. Fundamental distribution factor.

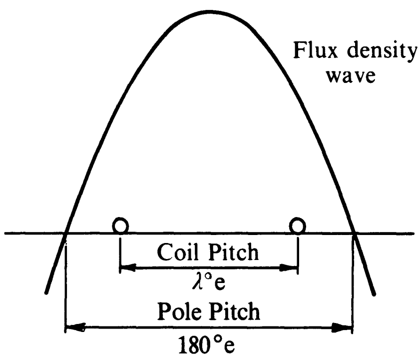


Fig. 3.6. Fundamental pitch factor.

The expression for the induced voltage given in (3.33) can then be written

$$E = 4{\cdot}44 k_d \Phi_m N f. \tag{3.40}$$

When the coil pitch is less than a pole pitch (180°e), it follows that the flux linking the coil and hence the induced voltage will be less than that for a full-pitch coil. For the conditions shown in Fig. 3.6 the coil pitch λ is less than the pole pitch and this effect can be accounted for by introducing the pitch factor k_p such that

$$k_p = \frac{\int_{-\lambda/2}^{\lambda/2} \cos\frac{p\theta}{2}\, d\left(\frac{p\theta}{2}\right)}{\int_{-\pi/2}^{\pi/2} \cos\frac{p\theta}{2}\, d\left(\frac{p\theta}{2}\right)} = \frac{\left[\sin\frac{p\theta}{2}\right]_0^{\lambda/2}}{\left[\sin\frac{p\theta}{2}\right]_0^{\pi/2}}.$$

That is,

$$k_p = \sin\frac{\lambda}{2}. \tag{3.41}$$

The winding factor k_w is given by

$$k_w = k_d k_p \tag{3.42}$$

and the general form of the induced voltage is

$$E = 4{\cdot}44 k_w \Phi_m N f. \tag{3.43}$$

3.4. Production of torque

The instantaneous value of the gross output power p_g from an electrical motor can be written

$$P_g = e i_a \tag{3.44}$$

where e is the generated e.m.f. and i_a the armature current.

For the case of an a.c. motor with a sinusoidally distributed flux, it follows from (3.32) that

$$e = (\omega_s N_a \Phi_m \cos \omega_s t) k_w$$

where N_a is the effective number of armature turns in series and k_w is the winding factor.

The armature current i_a can be written

$$i_a = I_{m_a} \sin(\omega_s t - \lambda)$$

where $90 - \lambda$ is the phase angle between e and i_a. Then

$$P_g = \omega_s (N_a k_w \Phi_m) I_{m_a} \cos \omega_s t \sin(\omega_s t - \lambda). \tag{3.45}$$

Then the electromagnetic torque produced by the machine is given by

$$t_e = \frac{P_g}{\omega} = \frac{p}{2}\frac{P_g}{\omega_s}$$

so that $$t_e = \frac{p}{2} k_w (N_a I_{ma})(\Phi_m) \cos \omega_s t \sin (\omega_s t - \lambda). \quad (3.46)$$

The maximum value of the flux, Φ_m, can be written in terms of the maximum value of the field m.m.f., F_1, as

$$\Phi_m = B_m\left(\frac{2}{p} dl\right) = \mu_0 H \frac{2dl}{p} = \frac{\mu_0 2dl}{gp} F_1$$

where d is the mean diameter at the air-gap, l is the length of core, and g is the air-gap length.

Equation (3.46) then becomes

$$t_e = \frac{p}{2} \frac{\mu_0 2dl}{gp} k_w F_1 F_2 \cos \omega_s t \sin (\omega_s t - \lambda)$$

where $F_2 = N_a I_{m_a}$, or

$$t_e = \frac{\mu_0 dl}{g} k_w \frac{F_1 F_2}{2} [\sin (2\omega_s t - \lambda) - \sin \lambda]. \quad (3.47)$$

The average value T_e is given by

$$T_e = -\frac{\mu_0 dl}{g} k_w \frac{F_1 F_2}{2} \sin \lambda. \quad (3.48)$$

The field flux density B_m is given by $B_m = \mu_0 F_1/g$, so that

$$T_e = -\frac{dl}{2} k_w B_m F_2 \sin \lambda \quad (3.49)$$

In the case of a d.c. machine the generated e.m.f. E is given from (3.35) as

$$E = \frac{pz}{2\pi a} \Phi \omega.$$

Then $$P_g = EI_a = \frac{pz}{2\pi a} \Phi I_a \omega. \quad (3.50)$$

It follows that the torque T_e is given by

$$T_e = \frac{P_g}{\omega} = \frac{pz}{2\pi a} \Phi I_a \quad (3.51)$$

or $$T_e = K\Phi I_a \quad (3.52)$$

where $K = pz/2\pi a$.

The general analysis given here has assumed that the rotating armature m.m.f. is sinusoidally distributed in space. In practice, the presence of m.m.f. harmonics must be taken into account.

Tutorial Problems

1. A 6-pole, double-layer, d.c. armature winding in 28 slots has 5 turns per coil. If the field flux is 0·03 Wb per pole and the speed of the rotor is 1000 r.p.m., find the value of the induced e.m.f. when the winding is (a) lap connected, (b) wave connected.

(*Answer:* 140 V; 420 V)

2. A 3-phase, 4-pole, star-connected synchronous machine, driven at 1500 r.p.m. has a 48-slot stator carrying a 60° spread double-layer winding with 83·3% pitch coils. There are 10 conductors in each slot. If the maximum value of the sinusoidally distributed field flux is 0·088 Wb calculate the fundamental distribution and pitch factors and the phase and line r.m.s. values of the generated e.m.f.

(*Answer:* 0·958; 0·966; 1450 V; 2510 V)

3. A certain electromechanical device has a magnetism characteristic given approximately by:

$$\Phi = \frac{kF}{a + F}\,\text{Wb}, \qquad k = (b + cx)$$

where x is the displacement. By considering both energy and co-energy in turn, show that the force expressions derived from either of these two methods of approach are identical.

Also find the mechanical work done if x is permitted to change slowly from zero by an amount b/c with exciting m.m.f. value F.

$$\left(\textit{Answer: } C\left(F - a \log_e \frac{a + F}{a}\right);\ bF - a \log_e \frac{a + F}{a}\right)$$

4. The stator and rotor of a machine each carry a single winding with the rotor short-circuited. The stator winding has a self inductance of 3 H, the corresponding value for the rotor is 2 H and the mutual inductance between the windings is given by 2·83 cos θ H where θ is the angle between the winding axes. All resistances can be neglected. Derive an expression for the torque as a function of θ when the machine is stationary when the stator current $i_s = 1{\cdot}41 \sin 100\,\pi t$. If the rotor is held with $\theta = 45°$e and then released, find the corresponding value of θ.

(*Answer:* $2 \sin 2\theta(1 - \cos 200t)$)

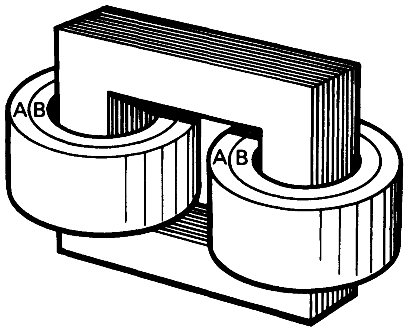
5. A 4-pole, d.c. series motor has a lap connected, double-layer armature winding with a total of 400 conductors. Calculate the gross torque developed for a flux per pole of 0·025 Wb and an armature current of 40 A.

(*Answer:* 63·6 N.m)

6. A 60-slot stator is to be wound with a 3-phase, 10-pole, double-layer, 60° spread winding with (a) full-pitch coils, (b) 5/6 pitch coils. Plot the total m.m.f. distribution in space for the instants in time when the phase A current is a maximum and when the phase A current is zero.

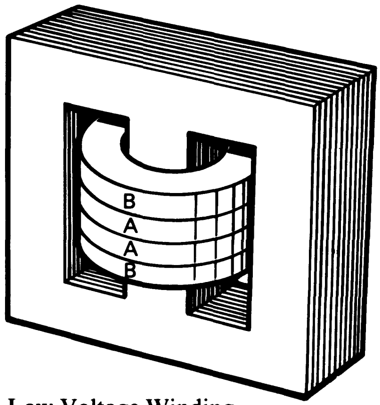
4. The transformer

The action of a transformer is a particular case of the principle of mutual inductance, and a transformer consists essentially of two windings, the primary and secondary on a common magnetic core. A transformer will be of either core-type or shell-type construction, and typical forms of construction are shown in Figs. 4.1 and 4.2. It can be seen from Figs. 4.1 and 4.2 that



A High Voltage Winding
B Low Voltage Winding

Fig. 4.1. Core-type construction.



B Low Voltage Winding
A High Voltage Winding

Fig. 4.2. Shell-type construction.

for a core-type construction the primary and secondary windings are wound as a pair of concentric coils on each limb, whereas for a shell-type construction the primary and secondary windings form interleaved layers on a single limb. In all cases, the core will be of laminated construction in order to reduce iron losses to a minimum.

4.1. The ideal single-phase transformer

When developing the theory of a single-phase transformer it is first convenient to consider the operation of the so-called 'ideal transformer', for which all the flux links all the turns on both windings; winding resistance and iron losses can be neglected and negligible magnetizing current is required to set up the flux.

An elementary diagram for an ideal transformer is shown in Fig. 4.3. When an alternating voltage $\mathbf{V}_1$ volts at frequency f Hz is applied to the primary windings, a mutual flux $\mathbf{\Phi}$ weber is set up which links both windings, and primary and secondary e.m.f.s $\mathbf{E}_1$ and $\mathbf{E}_2$ are set up such that

$$\mathbf{E}_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\mathbf{\Phi}}{dt}, \qquad \mathbf{E}_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\mathbf{\Phi}}{dt} \tag{4.1}$$

where N_1 and N_2 are, respectively, the primary and secondary turns.

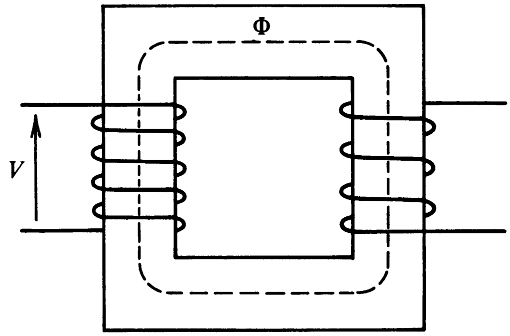


Fig. 4.3. Elementary diagram for ideal transformer.

It can be seen from (4.1) that the e.m.f.s $\mathbf{E}_1$ and $\mathbf{E}_2$ are in phase and displaced from $\mathbf{\Phi}$ by 90°. The primary applied voltage $\mathbf{V}_1$ is then equal to the primary e.m.f. $\mathbf{E}_1$, and the secondary terminal voltage $\mathbf{V}_2$ is equal to and in phase with the secondary e.m.f. $\mathbf{E}_2$.

If the mutual flux is assumed to vary sinusoidally with the supply frequency f Hz, the equation for the e.m.f. $\mathbf{E}_1$ can be written

$$\mathbf{E}_{m_1} = N_1 \frac{d}{dt} \Phi_m \sin 2\pi f t \tag{4.2}$$

where $\mathbf{E}_{m_1}$ is the peak value of the primary e.m.f. and Φ_m is the peak value of the mutual flux, then

$$\mathbf{E}_{m_1} = N_1 2\pi f \Phi_m \cos 2\pi f t.$$

Thus the r.m.s. value $\mathbf{E}_1$ of the primary e.m.f. will be given by

$$\mathbf{E}_1 = \frac{1}{\sqrt{2}} 2\pi f N_1 \Phi_m \tag{4.3}$$

or

$$\mathbf{E}_1 = 4{\cdot}44 f N_1 \Phi_m.$$

In a similar manner, the r.m.s. value $\mathbf{E}_2$ of the secondary e.m.f. will be given by

$$\mathbf{E}_2 = 4{\cdot}44 f N_2 \Phi_m. \tag{4.4}$$

When a load resulting in a secondary current $\mathbf{I}_2$ is connected to the transformer, an m.m.f. $N_2 I_2$ is set up. If the primary applied voltage is maintained constant, the primary e.m.f. and hence the resultant flux $\mathbf{\Phi}$ must be constant. Thus the secondary m.m.f. $N_2 I_2$ must be balanced by a primary m.m.f. $N_1 I_{21}$ set up by a primary current $\mathbf{I}_{21}$, such that $N_2 I_2 = N_1 I_{21}$.

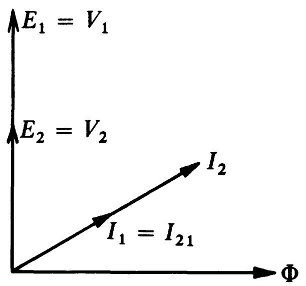


Fig. 4.4. Phasor diagram of ideal transformer.

The primary current $\mathbf{I}_{21}$ is then the primary reflection of the secondary current $\mathbf{I}_2$ and is known as the load component of the primary current or as the secondary current *referred* to the primary. A phasor diagram representing the operation of an ideal transformer can then be drawn as shown in Fig. 4.4.

4.2. The practical single-phase transformer

Consider now a practical transformer with the secondary on open-circuit. A magnetizing current $\mathbf{I}_m$ will be required to set up the mutual flux $\mathbf{\Phi}$ and this current will, under linear conditions, be in phase with the flux $\mathbf{\Phi}$. A practical transformer will obviously have iron losses in the core and a core-loss component of current $\mathbf{I}_c$ will be required to provide the power associated with these losses. This current will be in phase with the primary e.m.f. The no-load current $\mathbf{I}_0$ of the transformer will then be the phasor sum of $\mathbf{I}_m$ and $\mathbf{I}_c$. A phasor diagram representing this condition is shown in Fig. 4.5.

It has previously been assumed that the windings of the transformer have no resistance or leakage inductance. Since the windings consist of copper conductors it immediately follows that both the primary and secondary will have winding resistance. The total flux linking the separate windings can, for analytical purposes, be divided into two components, the mutual flux which links both windings and the leakage flux linking one winding only. The relevant flux paths are illustrated in Fig. 4.6. The leakage paths are mainly through air and the effect is to introduce leakage inductances L_1 and L_2 into the primary and secondary, respectively, given in each case by the leakage flux linkage per unit current. Each leakage inductance will result in a leakage reactance.

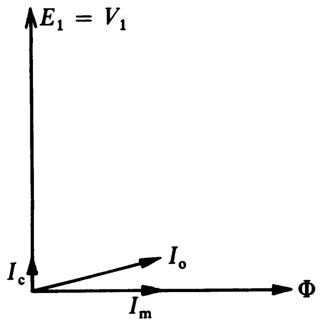


Fig. 4.5. Phasor diagram with secondary on open-circuit.

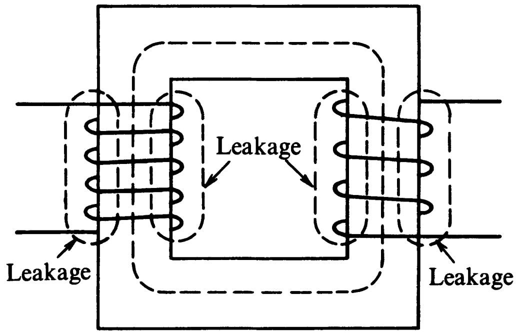


Fig. 4.6. Main and leakage flux paths.

Consider now the practical transformer on load. The total primary current $\mathbf{I}_1$ must now meet two requirements. It must supply the no-load current $\mathbf{I}_0$ of the transformer and supply a current $\mathbf{I}_{21}$ to counteract the demagnetizing effect of the secondary load current $\mathbf{I}_2$. The voltage $\mathbf{V}_1$ applied to the primary winding can then be equated to the phasor sum of the voltage drops in the primary resistance R_1 and the primary leakage reactance X_1 and the primary e.m.f. $\mathbf{E}_1$. The no-load current $\mathbf{I}_0$ can be resolved into two components, the magnetizing current $\mathbf{I}_m$ in phase with the mutual flux and hence in quadrature with the e.m.f. and the core-loss current $\mathbf{I}_c$ in phase with the e.m.f. In an equivalent circuit, the effect of the no-load current can be simulated by means of a shunt branch of a non-inductive conductance g in parallel with an inductive susceptance b. Thus an equivalent circuit representing the primary of a practical transformer on load is shown in Fig. 4.7. In the parallel combination shown, the power $E_1{}^2g$ accounts for the iron loss and the current $\mathbf{I}_m = \mathbf{E}_1 b$

is the magnetizing current. Both g and b are usually determined at rated primary voltage and frequency and are then assumed to remain constant for the small departures from rated conditions associated with normal operation.

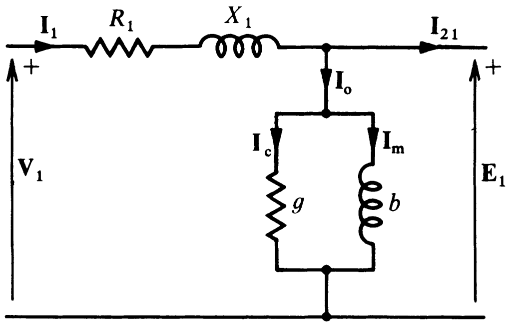


Fig. 4.7. Equivalent circuit for primary with secondary on load.

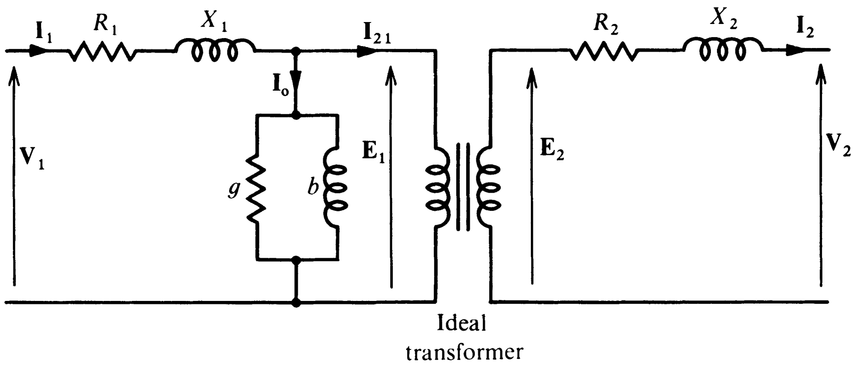


Fig. 4.8. Practical transformer on load: (a) Equivalent circuit.

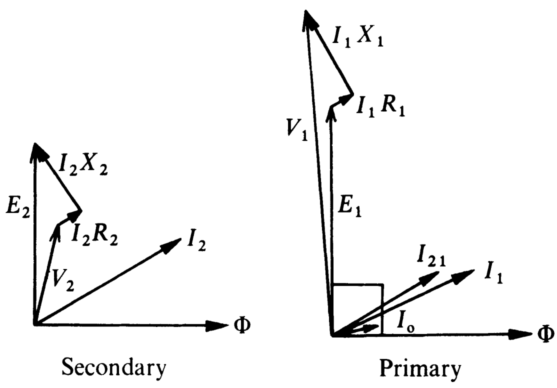


Fig. 4.8(b) Phasor diagram.

The transformation ratio is taken into account by introducing an ideal transformer into the equivalent circuit such that $E_1/N_1 = E_2/N_2$. The secondary e.m.f. $\mathbf{E}_2$ can then be equated to the phasor sum of the secondary terminal voltage $\mathbf{V}_2$ and the voltage drops in the secondary resistance R_2 and leakage reactance X_2. An equivalent circuit representing a practical transformer on load is shown in Fig. 4.8(a) and the corresponding phasor diagram in Fig. 4.8(b). It can be seen from Fig. 4.8(a) that the equivalent circuit for a practical transformer is that of an ideal transformer plus external impedances.

4.3. Derivation of equivalent circuits

It is, in general, not convenient to base calculations on a circuit which includes an ideal transformer, and the process of removing the ideal transformer is known as 'referring' all quantities to one side (either the primary or secondary) of the transformer.

Consider the secondary voltage equation,

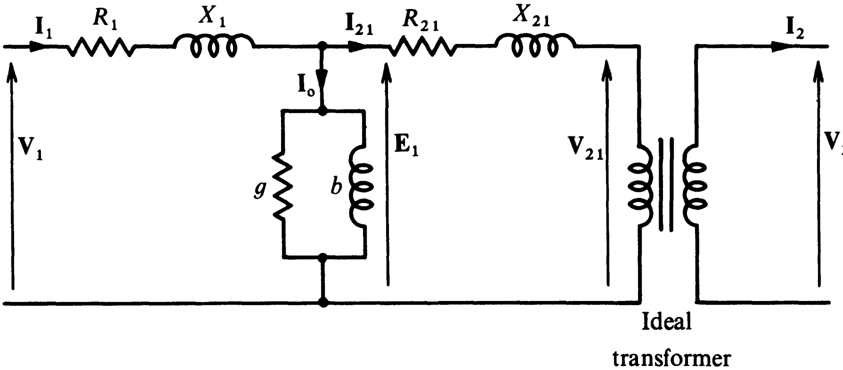
$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2(R_2 + jX_2). \tag{4.5}$$

Since $$\mathbf{E}_2 = \mathbf{E}_1 \frac{N_2}{N_1} = \frac{\mathbf{E}_1}{N}, \quad \text{and} \quad \mathbf{I}_2 = N\mathbf{I}_{21},$$

equation (4.5) can be written in the form

$$\frac{\mathbf{E}_1}{N} = \mathbf{V}_2 + N\mathbf{I}_{21}(R_2 + jX_2), \quad \text{or} \quad \mathbf{E}_1 = N\mathbf{V}_2 + \mathbf{I}_{21}(N^2R_2 + jN^2X_2).$$

Thus the equivalent circuit of Fig. 4.8(a) can be redrawn as shown in Fig. 4.9 in which $R_{21} = N^2R_2$ and $X_{21} = N^2X_2$ are, respectively, the secondary resistance and leakage reactance referred to the primary, and $\mathbf{V}_{21} = N\mathbf{V}_2$ is the secondary terminal voltage referred to the primary. The circuit of Fig. 4.9



$$R_{2_1} = N^2R_2;\; X_{2_1} = N^2X_2$$

Fig. 4.9. Equivalent circuit with referred values.

is usually drawn with the ideal transformer omitted as shown in Fig. 4.10, and this circuit is usually known as the exact equivalent circuit. Various approximations to the exact equivalent circuit can be made and the usual

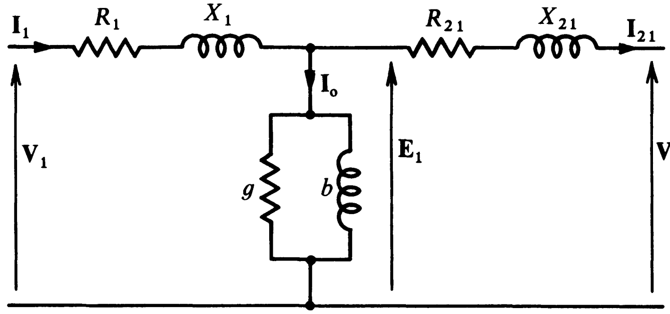
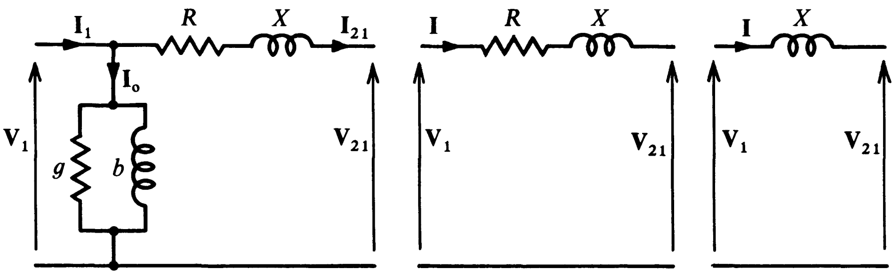


Fig. 4.10. Exact equivalent circuit.

approximate equivalent circuit for small transformers is shown in Fig. 4.11(a). This circuit differs from the exact equivalent circuit in that the small voltage drop produced by the no-load current in the primary impedance has been neglected. For a medium-size power transformer the no-load current can usually be neglected, and for large power transformers the winding resistance can also be neglected. The forms of equivalent circuit for such transformers is shown in Figs. 4.11(b) and (c).

The complete operating performance of any single-phase transformer can then be evaluated on the basis of the appropriate equivalent circuit. It is usual to base calculations on one of the circuits of Fig. 4.11, which have the advantage that the parameters can be determined by two simple tests, whereas determination of the component resistances and leakage reactances for use in an exact equivalent circuit can be difficult.



$$R = R_1 + R_{21}; \qquad X = X_1 + X_{21}$$

(a) (b) (c)

Fig. 4.11. Forms of approximate equivalent circuits:
(a) Small transformer.
(b) Power transformer.
(c) Large power transformer.

4.4. Determination of transformer parameters

The parameters on the equivalent circuit can, of course, be obtained from design considerations. There are, however, two simple tests, the open-circuit and short-circuit tests, which can be carried out to determine the circuit parameters.

(*a*) *Open-circuit test*

The high-voltage winding is on open-circuit and the low-voltage winding connected to a variable voltage supply, at normal frequency as shown in Fig. 4.12, in which the primary is assumed to be the low-voltage winding.

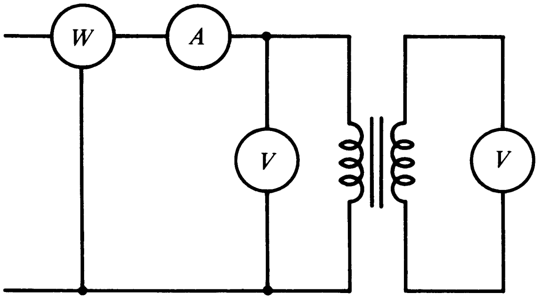


Fig. 4.12. Connections for open-circuit test.

The input current and power and the voltage across the open-circuited winding are measured for a range of applied voltages up to 125% of the rated voltage. Then, at rated voltage,

Iron loss $P_i = V^2 g$, i.e., $g = P_i/V^2$.
Open-circuit admittance $y = I/V$.
Magnetizing susceptance $b = \sqrt{(y^2 - g^2)}$.

In these tests the small copper loss in the primary resistance has been neglected.

If this test is performed at two different frequencies f_a and f_b and the input power plotted to a base of input voltage divided by frequency, V/f, as shown in Fig. 4.13, the iron losses can be separated into the component hysteresis and eddy current losses. At a given flux density, hysteresis loss is proportional to frequency and eddy current loss is proportional to the square of frequency. Now fixed flux density corresponds to a fixed voltage–frequency ratio and for the ordinate LMN in Fig. 4.13,

$$P_a = Af_a + Bf_a^2 \text{ at frequency } f_a, \qquad P_b = Af_b + Bf_b^2 \text{ at frequency } f_b$$

where P_a and P_b are the total iron losses at frequencies f_a and f_b respectively and A and B are constants.

The constants A and B can now be determined and the component iron losses at any frequency for this fixed value of flux density can be determined. By considering several different values of flux density, the component iron losses at any value of flux density can be found.

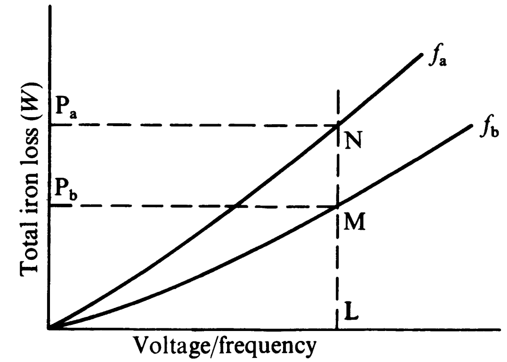


Fig. 4.13. Separation of iron losses.

(b) Short-circuit test

The low-voltage winding is short-circuited through an ammeter and the high-voltage winding is connected to a variable voltage supply at normal frequency as shown in Fig. 4.14 in which it is assumed that the primary is the high-voltage winding. The input current and power and the voltage across the high-voltage winding are measured for a range of short-circuit currents up to 125 per cent of the rated current. The input current is made up of the reflection of the short-circuited current and the no-load current and the input power is made up of copper loss, iron loss, and dielectric loss.

Copper loss $P_c = I^2R$ and Total resistance $R = P_c/I^2$.
Short-circuit impedance $Z = V/I$.
Total leakage reactance $X = \sqrt{(Z^2 - R^2)}$.

Thus the total resistance and total leakage reactance referred to the high-voltage winding can be found.

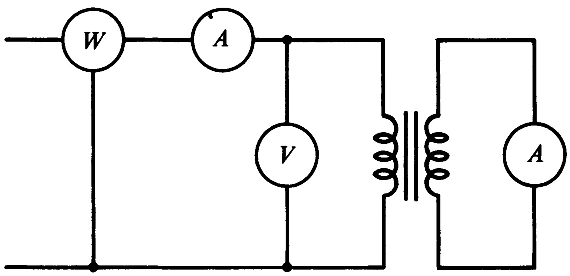


Fig. 4.14. Connections for short-circuit test.

EXAMPLE 4.1. The results of the open-circuit and short-circuit tests of a 230/100 V 10 kVA single-phase transformer are as follows:

	Primary	*Secondary*
Open-circuit test	Open-circuit	100 V, 6 A, 154 W
Short-circuit test	18 V, 43·5 A, 240 W	Short-circuit

Determine the parameters of the approximate equivalent circuit of Fig. 4.11(a) and express these quantities in per-unit values.

Solution: From short-circuit test

$$R = R_1 + R_{21} = \frac{P}{I^2} = \frac{240}{43{\cdot}5^2} = 0{\cdot}127\ \Omega$$

$$Z = \frac{V}{I} = \frac{18}{43{\cdot}5} = 0{\cdot}414\ \Omega.$$

Then $$X = X_1 + X_{21} = \sqrt{(Z^2 - R^2)} = 0{\cdot}394\ \Omega.$$

Since the open-circuit test was performed with the primary on open-circuit, the results of this test must be referred to the primary if parameters applicable to an equivalent circuit referred to the primary are to be obtained. Then results of open-circuit test become

$$100 \times \frac{230}{100} = 230\ \text{V}, \qquad 6 \times \frac{100}{230} = 2{\cdot}61\ \text{A},\ 154\ \text{W}.$$

Thus $$g = \frac{P}{V^2} = \frac{154}{230^2} = 0{\cdot}00291\ \text{mho}$$

$$y = \frac{I}{V} = \frac{2{\cdot}61}{230} = 0{\cdot}01136\ \text{mho}, \qquad b = \sqrt{(y^2 - g^2)} = 0{\cdot}01094\ \text{mho}.$$

Then the approximate equivalent circuit can be drawn. Now,

$$Z_{pu} = \frac{Z}{Z_b} \quad \text{and} \quad Z_b = \frac{V_r}{I_r} = \frac{V_r^2}{(VA)_r} = \frac{230^2}{10^4} = 5{\cdot}28.$$

Then $$R_{pu} = \frac{R}{Z_b} = \frac{0{\cdot}127}{5{\cdot}28} = 0{\cdot}024, \qquad X_{pu} = \frac{X}{Z_b} = \frac{0{\cdot}394}{5{\cdot}28} = 0{\cdot}0784.$$

Similarly, $$y_b = \frac{I_r}{V_r} = \frac{1}{Z_b} = 0{\cdot}189.$$

Then

$$g_{pu} = \frac{g}{y_b} = \frac{0{\cdot}00291}{0{\cdot}189} = 0{\cdot}0154, \qquad b_{pu} = \frac{b}{y_b} = \frac{0{\cdot}01094}{0{\cdot}189} = 0{\cdot}058.$$

4.5. Performance calculations

It should be noted at this stage that, in practice, the primary will be supplied at rated voltage and a particular load current at a particular power factor will be supplied from the secondary. If it is assumed that the parameters of the equivalent circuit are known, calculations based on the approximate equivalent circuit of Fig. 4.11(a) can become complicated. A phasor diagram corresponding to the circuit of Fig. 4.11(a) is shown in Fig. 4.15 in which the quantities $\mathbf{V}_1$, R, X, g, b, $\mathbf{I}_{21}$, and ϕ are assumed to be known. It should be noted, at this stage, that Fig. 4.15 is drawn for a lagging power-factor and

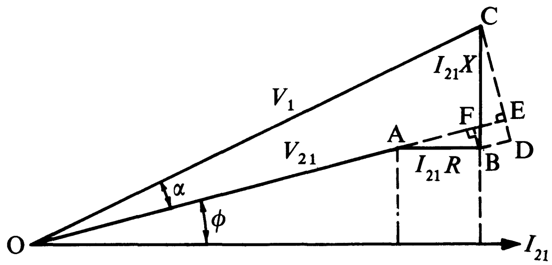


Fig. 4.15. Phasor diagram for approximate equivalent circuit.

under these conditions the power angle ϕ is assumed to be negative. Then by a simple application of Pythagoras's theorem

$$V_1{}^2 = [V_{21}\cos(-\phi) + I_{21}R]^2 + [V_{21}\sin(-\phi) + I_{21}X]^2.$$

That is,

$$V_1{}^2 = V_{21}^2 + 2I_{21}V_{21}(R\cos\phi - X\sin\phi) + I_{21}^2(R^2 + X^2).$$

Now $$R = Z\cos\theta \quad\text{and}\quad X = Z\sin\theta$$

where $Z^2 = R^2 + X^2$, $\tan\theta = X/R$. Thus

$$V_1^2 = V_{21}^2 + 2I_{21}ZV_{21}(\cos\theta\cos\phi - \sin\theta\sin\phi) + I_{21}^2Z^2,$$

or $$V_1{}^2 = V_{21}^2 + 2I_{21}ZV_{21}\cos(\theta + \phi) + I_{21}^2Z^2 \tag{4.6}$$

where ϕ is negative for lagging power factors.

This is a quadratic equation which can be solved for V_{21}. The phasor diagram can then be drawn and the input current $\mathbf{I}_1$ and power-factor $\cos\phi_1$ found.

Some simplification can be made when the angle between $\mathbf{V}_1$ and $\mathbf{V}_{21}$ is small and (4.6) can be modified to the approximate volt drop equation. Referring to Fig. 4.15, OC $= V_1$ and, if α is small, OC $\simeq$ OE. Now

OE = OA + AF + FE. But OA = V_{21}, AF = $I_{21}R\cos(-\phi)$, FE = $I_{21}X\sin(-\phi)$. Then

$$V_1 = V_{21} + I_{21}(R\cos\phi - X\sin\phi),$$

or

$$V_{21} = V_1 - I_{21}(R\cos\phi - X\sin\phi).$$

Thus

$$V_{21} = V_1 - I_{21}Z\cos(\theta + \phi) \tag{4.7}$$

where ϕ is negative for lagging power-factors.

Both (4.6) and (4.7) can be rewritten directly in terms of per-unit (p.u.) quantities. In particular, if the transformer is operating on full load ($I_{21\text{pu}} = 1$) with rated primary voltage ($V_{21\text{pu}} = 1$), (4.6) can be written as

$$1 = V_{21\,\text{pu}}^2 + 2Z_{\text{pu}}V_{21\,\text{pu}}\cos(\theta + \phi) + Z_{\text{pu}}^2.$$

Then

$$2V_{21\,\text{pu}} = -2Z_{\text{pu}}\cos(\theta + \phi) \pm [4Z_{\text{pu}}^2\cos^2(\theta + \phi) - 4(Z_{\text{pu}}^2 - 1)]^{\frac{1}{2}},$$

that is,

$$V_{21\,\text{pu}} = -Z_{\text{pu}}\cos(\theta + \phi) \pm [1 - Z_{\text{pu}}^2\sin^2(\theta + \phi)]^{\frac{1}{2}}.$$

The only practical solution is

$$V_{21\,\text{pu}} = [1 - Z_{\text{pu}}^2\sin^2(\theta + \phi)]^{\frac{1}{2}} - Z_{\text{pu}}\cos(\theta + \phi). \tag{4.8}$$

In a similar manner, (4.7) can be written

$$V_{21\,\text{pu}} = 1 - Z_{\text{pu}}\cos(\theta + \phi). \tag{4.9}$$

Comparison of (4.8) and (4.9) shows that the approximate volt drop equation is exact when $Z_{\text{pu}}^2\sin^2(\theta + \phi) = 0$. This can only be true in practice when $\theta + \phi = 0$ or 180°.

EXAMPLE 4.2. A 10 kVA, 2000/400 V, single-phase transformer has resistances and leakage reactances as follows:

$$R_1 = 5{\cdot}2\,\Omega;\quad X_1 = 12{\cdot}5\,\Omega;\quad R_2 = 0{\cdot}2\,\Omega;\quad X_2 = 0{\cdot}5\,\Omega.$$

Determine the value of the secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with power-factor 0·8 lag. The no-load current can be neglected.

Solution: The parameters of the equivalent circuit referred to the primary are

$$R = R_1 + N^2R_2 = 5{\cdot}2 + 5^2 \times 0{\cdot}2 = 10{\cdot}2\,\Omega$$

$$X = X_1 + N^2X_2 = 12{\cdot}5 + 5^2 \times 0{\cdot}5 = 25\,\Omega.$$

Then

$$Z = \sqrt{(R^2 + X^2)} = \sqrt{(10{\cdot}2^2 + 25^2)} = 27\ \Omega,$$

$$\theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{25}{10{\cdot}2} = 67{\cdot}8^\circ.$$

Now

$$Z_b = \frac{V_r}{I_r} = \frac{V_r^2}{S_r} = \frac{2000^2}{10 \times 10^3} = 400\ \Omega, \qquad Z_{pu} = \frac{Z}{Z_b} = \frac{27}{400} = 0{\cdot}0675.$$

Given $V_{1\,pu} = 1$ and $I_{pu} = 1$, $\cos\phi = 0{\cdot}8$ lag, and $\phi = -36{\cdot}8^\circ$, (4.8) can be written as

$$V_{21\,pu} = [1 - 0{\cdot}0675^2 \sin^2(67{\cdot}8 - 36{\cdot}8)]^{\frac{1}{2}} - 0{\cdot}0675\cos(67{\cdot}8 - 36{\cdot}8)$$
$$= 0{\cdot}9415.$$

$$\text{Secondary voltage } V_2 = 0{\cdot}9415 \times 400 = 376{\cdot}6\ \text{V}.$$

If the approximate voltage drop equation (4.9) is used,

$$V_{21} = 1 - 0{\cdot}0675\cos(67{\cdot}8 - 36{\cdot}8) = 0{\cdot}9421\ \text{V}.$$

$$\text{Secondary voltage } V_2 = 0{\cdot}9421 \times 400 = 376{\cdot}8\ \text{V}.$$

It can immediately be seen that, in this case, the error involved in using the approximate equation is negligible.

4.6. Voltage regulation

The voltage regulation is defined as the rise in the secondary terminal voltage when full load is removed with constant primary applied voltage. Then, from (4.9)

$$\text{p.u. regulation} = 1 - V_{21\,pu} = 1 + Z_{pu}\cos(\theta + \phi) - [1 - Z_{pu}^2 \sin^2(\theta + \phi)]^{\frac{1}{2}} \quad (4.10)$$

and, from the approximate volt drop, equation (4.9),

$$\text{p.u. regulation} = 1 - V_{21\,pu} = Z_{pu}\cos(\theta + \phi). \quad (4.11)$$

Conditions for maximum and zero regulation can be obtained directly from (4.10) and (4.11).

From (4.10), zero regulation occurs when

$$0 = 1 + Z_{pu}\cos(\theta + \phi) - [1 - Z_{pu}^2 \sin^2(\theta + \phi)]^{\frac{1}{2}},$$

i.e.,

$$1 + 2Z_{pu}\cos(\theta + \phi) + Z_{pu}^2\cos^2(\theta + \phi) = 1 - Z_{pu}^2\sin^2(\theta + \phi),$$

or

$$Z_{pu}^2 + 2Z_{pu}\cos(\theta + \phi) = 0.$$

Hence $$Z_{pu} = 0 \quad \text{or} \quad \cos(\theta + \phi) = -\frac{Z_{pu}}{2}. \tag{4.12}$$

In a similar manner, from the approximate equation (4.11), the regulation will be zero when $Z_{pu} \cos(\theta + \phi) = 0$, i.e.,

$$Z_{pu} = 0 \quad \text{or} \quad \cos(\theta + \phi) = 0. \tag{4.13}$$

The first condition ($Z_{pu} = 0$) in (4.12) and (4.13) will not be considered further. Comparison of the second condition in (4.12) and (4.13) shows that, if Z_{pu} is relatively large, considerable error can be involved in using the approximate voltage drop equation for the calculation of the condition for zero regulation.

Maximum regulation will always occur at large values of the angle ϕ and under these conditions the approximate volt drop equation can be used. Then, from (4.11) directly, maximum regulation will occur when

$$\cos(\theta + \phi) = 1. \tag{4.14}$$

Thus maximum regulation will occur when the lagging power angle ϕ is numerically equal to the impedance angle θ of the transformer.

4.7. Efficiency

Since the transformer is a static piece of apparatus, the losses are limited to:

(1) Copper loss in the resistance of the windings which is variable with load current.
(2) Iron loss in the core made up of component hysteresis and eddy current losses which are usually considered to be constant.
(3) Stray loss produced by stray flux producing eddy current losses in the conductors.
(4) Dielectric loss in the insulating material which is appreciable only in the particular case of high-voltage transformers.

In general, efficiency can be written in terms of any two of input power, output power, and losses. That is,

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}.$$

For the approximate equivalent circuit of Fig. 4.11(a)

$$\text{Efficiency p.u.} = \frac{V_{21\,pu} I_{21\,pu} \cos\phi}{V_{21\,pu} I_{21\,pu} \cos\phi + I_{21\,pu}^2 R_{pu} + V_{1\,pu}^2 g_{pu}}. \tag{4.15}$$

When (4.15) is differentiated with respect to the load current $I_{21\,pu}$ and the resulting equation equated to zero, it follows that maximum efficiency occurs when

$$I_{21\,pu}^2 R_{pu} = V_{1\,pu}^2 g_{pu} \tag{4.16}$$

i.e., when the variable loss equals the fixed loss. Then, with the primary supplied at rated voltage,

$$\text{Maximum efficiency p.u.} = \frac{V_{21\,\text{pu}} I_{21\,\text{pu}} \cos\phi}{V_{21\,\text{pu}} I_{21\,\text{pu}} \cos\phi + 2g_{\text{pu}}}. \tag{4.17}$$

The maximum efficiency for any power-factor occurs at the same load, and the highest possible efficiency occurs at unity power-factor.

A transformer which is to operate continuously on full load will be designed to have maximum efficiency on full load. In the case of transformers used for distribution purposes which are to operate for long periods on light load, the point of maximum efficiency would be arranged to be between one-half and three-quarters of full load.

If it is required to measure the efficiency of a transformer on load, it is normally inconvenient to dissipate the rated output of the transformer in

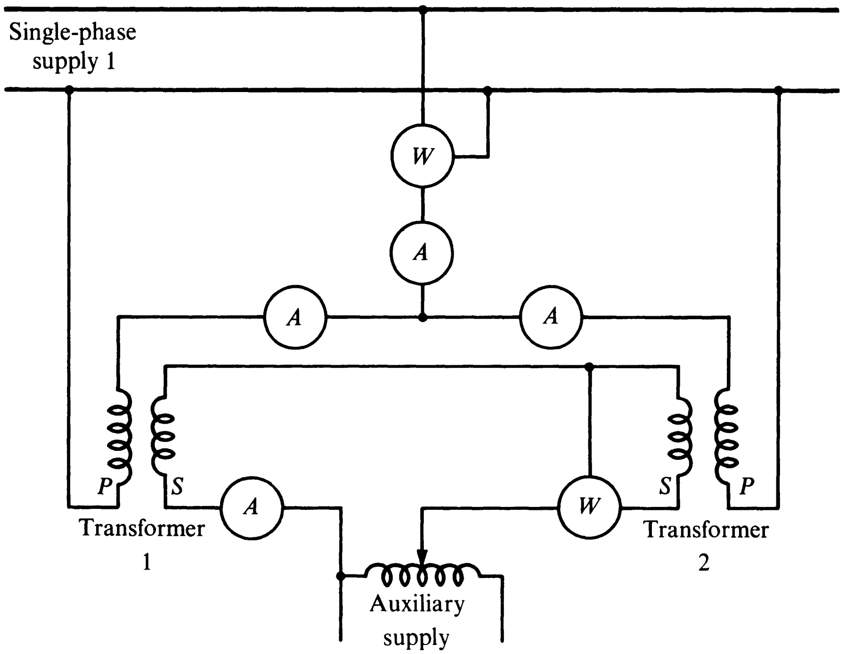


Fig. 4.16. Back-to-back test.

load banks, and a method of testing known as 'phantom load' testing can be used. For the particular case of the transformer, this method is known as the Back-to-Back or Sumpner Test and requires two identical transformers. The circuit used is shown in Fig. 4.16 in which the primary windings are connected in parallel and supplied at normal voltage and frequency, and the secondary windings are connected in series opposition and supplied through a variable voltage regulator. Then, as the voltage applied to the secondary winding is increased, the phantom load on the transformer increases. Under these conditions, the wattmeter W_1 in the primary circuit will record the total iron loss and the wattmeter W_2 in the secondary circuit will record the total copper loss. If facilities for temperature measurement are included, a heat run can also be performed and the final temperature rise of the transformer on load can be obtained.

4.8. Parallel operation

Transformers are said to be connected in parallel when their primary windings are connected to a common voltage supply and their secondary windings are connected to a common load. If the two transformers are to operate satisfactorily in parallel, the turns ratio of the two transformers should be the same and only points at the same potential should be joined together.

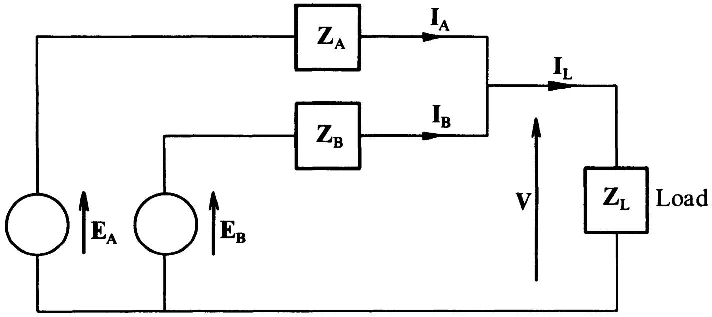


Fig. 4.17. Equivalent circuit for parallel operation.

An equivalent circuit, with all quantities referred to the secondary, representing the parallel operation of two transformers, A and B, is shown in Fig. 4.17. Then, by inspection,

$$\mathbf{V} = \mathbf{E}_A - \mathbf{I}_A\mathbf{Z}_A = \mathbf{E}_B - \mathbf{I}_B\mathbf{Z}_B \quad (4.18)$$

and

$$\mathbf{I}_A + \mathbf{I}_B = \mathbf{I}_L. \quad (4.19)$$

Consider the case of parallel operation of two transformers A and B with equal open-circuit voltages so that, in equation (4.18), $\mathbf{E}_A = \mathbf{E}_B$.

Then, from (4.18)

$$\mathbf{I}_A\mathbf{Z}_A = \mathbf{I}_B\mathbf{Z}_B.$$

If $\mathbf{I}_B$ is eliminated between (4.18) and (4.19) it follows that

$$\mathbf{I}_A = \mathbf{I}_L \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{I}_L \frac{1}{1 + \mathbf{Z}_A/\mathbf{Z}_B}. \quad (4.20)$$

In a similar manner

$$\mathbf{I}_B = \mathbf{I}_L \frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{I}_L \frac{1}{1 + \mathbf{Z}_B/\mathbf{Z}_A}. \tag{4.21}$$

If the load is specified in terms of kVA and not current, (4.20) and (4.21) can be written

$$\mathbf{S}_A = \mathbf{S}_T \frac{1}{1 + \mathbf{Z}_A/\mathbf{Z}_B} \tag{4.22}$$

and

$$\mathbf{S}_B = \mathbf{S}_T \frac{1}{1 + \mathbf{Z}_B/\mathbf{Z}_A} \tag{4.23}$$

where $\mathbf{S}_T = \mathbf{V}\mathbf{I}_L$ is the total load kVA, $\mathbf{S}_A$ is the kVA of transformer A, and $\mathbf{S}_B$ is the kVA of transformer B.

It is often convenient to specify the percentage or per-unit values of resistance and leakage reactance for a transformer, and in these circumstances (4.20) to (4.23) do not, in general, apply directly.

It has previously been shown that per-unit impedance $\mathbf{Z}_{pu}$ is given by $\mathbf{Z}_{pu} = \mathbf{Z}/Z_b$, where Z_b is a base impedance. Then

$$\mathbf{Z} = \mathbf{Z}_{pu} Z_b = \mathbf{Z}_{pu} \frac{V_r}{I_r} = \mathbf{Z}_{pu} \frac{V_r^2}{S_r},$$

where V_r is the rated voltage and S_r is the rated kVA. Then

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}} \frac{V_{rA}^2}{V_{rB}^2} \frac{S_{rB}}{S_{rA}}. \tag{4.24}$$

It has previously been assumed that the open-circuit voltages are equal and under these conditions (4.24) reduces to

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}} \frac{S_{rB}}{S_{rA}}. \tag{4.25}$$

It is immediately apparent from (4.25) that, if the transformers are to share the load in proportion to their ratings, their per-unit impedances must have the same magnitude and that, if the transformers are to operate at the same power-factor, their per-unit impedances must have the same phase angle.

When the load is specified in kVA some difficulty can arise in the calculation of the terminal voltage and the technique applicable to the solution of this problem is best illustrated by means of a numerical example.

EXAMPLE 4.3. A 500 kVA, 33/3·3 kV single-phase transformer with a resistance voltage drop of 1·5% and a reactance voltage drop of 6% is connected in parallel with a 1000 kVA, 33/3·3 kV single-phase transformer with a resistance voltage drop of 1% and a reactance voltage drop of 6·2%. Find the kVA loading and operating power-factor of each transformer when the total load is 1200 kVA at power-factor 0·8 lagging. If the transformer primaries are connected to constant frequency, 33 kV infinite busbars, calculate the load terminal voltage.

Solution: Transformer A, 33/3·3 kV, 500 kVA;

$$R_{pu} = 0{\cdot}015, \quad X_{pu} = 0{\cdot}06. \quad \therefore \ \mathbf{Z}_{A\,pu} = 0{\cdot}06185\underline{/76^\circ}.$$

Transformer B, 33/3·3 kV, 1000 kVA;

$$R_{pu} = 0{\cdot}01, \quad X_{pu} = 0{\cdot}062. \quad \therefore \ \mathbf{Z}_{B\,pu} = 0{\cdot}06201\underline{/80{\cdot}8^\circ}.$$

Now from (4.22),
$$\mathbf{S}_A = \frac{\mathbf{S}_T}{1 + \mathbf{Z}_A/\mathbf{Z}_B}.$$

From (4.25),

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}}\frac{S_{B_r}}{S_{A_r}} = \frac{0{\cdot}06185\underline{/76^\circ}}{0{\cdot}062\underline{/80{\cdot}8^\circ}}\frac{1000}{500},$$

$$= 1{\cdot}995\backslash 4{\cdot}8^\circ = 1{\cdot}988 - j0{\cdot}167.$$

Hence,
$$1 + \frac{\mathbf{Z}_A}{\mathbf{Z}_B} = 2{\cdot}988 - j0{\cdot}167 = 2{\cdot}993\backslash 3{\cdot}2^\circ.$$

Then, from (4.22)
$$\mathbf{S}_A = \frac{1200\backslash 36{\cdot}8}{2{\cdot}993\backslash 3{\cdot}2^\circ} = 402\backslash 33{\cdot}6^\circ.$$

Then $\mathbf{S}_A = 403$ kVA, $\cos\phi_A = \cos 33{\cdot}6 = 0{\cdot}833$ lag.

Now from (4.23)
$$\mathbf{S}_B = \frac{\mathbf{S}_T}{1 + \mathbf{Z}_B/\mathbf{Z}_A}$$

then
$$\frac{\mathbf{Z}_B}{\mathbf{Z}_A} = 1{\cdot}502 + j0{\cdot}0422 = 1{\cdot}503\underline{/1{\cdot}6^\circ},$$

and
$$\mathbf{S}_B = \frac{\mathbf{S}_T}{1 + \mathbf{Z}_B/\mathbf{Z}_A} = \frac{1200\backslash 36{\cdot}8}{1{\cdot}503\backslash 1{\cdot}6} = 798\backslash 38{\cdot}4.$$

Then $\mathbf{S}_B = 798$ kVA, $\cos\phi_B = \cos 38{\cdot}4^\circ = 0{\cdot}784$.

If the transformers are supplied at rated primary voltage the approximate voltage drop equation can be written as

$$1 = V_{21\,pu} + I_{pu}Z_{pu}\cos(\theta - \phi).$$

Then, for transformer A

$$1 = V_{pu} + I_{A\,pu} \times 0{\cdot}06185\cos(76 - 33{\cdot}6^\circ).$$

This equation contains two unknowns, but if both sides are multiplied by

V_{pu} the equation becomes

$$V_{pu} = V_{pu}^2 + V_{pu}I_{A\,pu} \times 0{\cdot}06185 \cos 32{\cdot}4^\circ.$$

Now

$$V_{pu}I_{A\,pu} = \frac{S_A}{S_{A_r}} = \frac{403}{500} = 0{\cdot}804.$$

Hence $V_{pu}^2 - V_{pu} + 0{\cdot}804 \times 0{\cdot}06185 \cos 32{\cdot}4^\circ = 0$. This is a quadratic in V_{pu} and can be solved to give $V_{pu} = 0{\cdot}996$ or $0{\cdot}004$. The value of $V_{pu} = 0{\cdot}004$ is obviously absurd and the load voltage V_L is given by

$$V_L = 3300 \times 0{\cdot}996 = 3287 \text{ V}.$$

4.9. The waveform of no-load current

It has previously been assumed that when the mutual flux and hence the e.m.f.s vary sinusoidally, the no-load current of the transformer will also vary sinusoidally. It is, however, common practice in transformer design to use high core flux densities which will, in general, lead to saturation of the core and a non-linear magnetization curve for the transformer. It immediately follows that if the flux is assumed to vary sinusoidally, the magnetizing current cannot vary sinusoidally. If, at this stage hysteresis loss is neglected, the waveform of the magnetizing current can be obtained if the magnetization curve for the transformer core is known and the method is best illustrated by means of a numerical example.

Consider a 50 Hz single-phase transformer with a 200 V primary winding with 260 turns. The effective cross-sectional area of the core is $2{\cdot}5 \times 10^{-3}$ m^2 and the mean length is 0·65 m. The magnetization curve is given in Fig. 4.18.

Now from the e.m.f. equation for a transformer

$$E = 4{\cdot}44 f N \Phi_m$$

and, if the primary leakage impedance is neglected on no-load,

$$200 = 4{\cdot}44 \times 50 \times 260 \times B_m \times 2{\cdot}5 \times 10^{-3} \quad \text{so that} \quad B_m = 1{\cdot}4 \text{ Wb/m}^2.$$

If the flux is assumed to vary sinusoidally it follows that the instantaneous value b of the flux density is given by

$$b = B_m \sin 2\pi f t = 1{\cdot}4 \sin 314{\cdot}2t = 1{\cdot}4 \sin \theta.$$

Then the corresponding instantaneous values in the magnetizing current can be obtained by the method shown in Table 4.1. For a particular instant in time, the instantaneous flux density can be found and the corresponding value of magnetizing force obtained from the magnetization curve plotted in Fig. 4.18. Thus the magnetizing current i_0 can be obtained from the value of h.

TABLE 4.1

Time (θ°e)	*Flux density* ($b = 1{\cdot}4 \sin \theta$)	*Magnetizing force* (h)	*Magnetizing current* ($i_0 = Hl/N$)
0	0	0	0
15	0·366	80	0·2
30	0·7	120	0·3
45	0·99	200	0·5
60	1·21	500	1·4
75	1·35	1072	2·68
90	1·4	1452	3·63

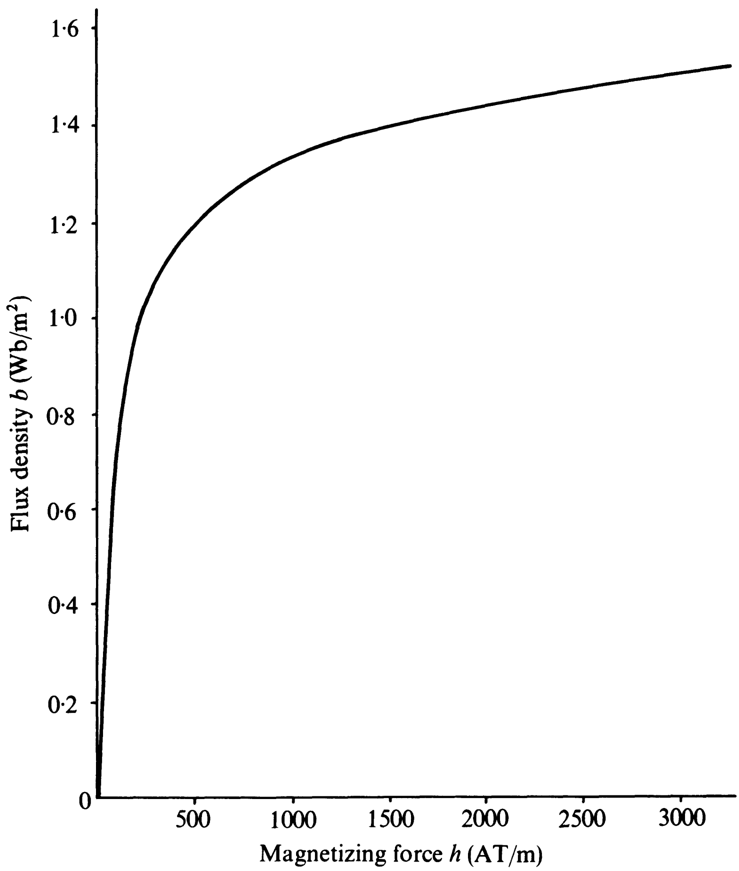


Fig. 4.18. Magnetization curve.

Since the waveform is symmetrical about $\theta = 90°$, values of current need only be obtained for one-quarter of a cycle. The waveform of flux density and magnetizing current are shown over one-half cycle in Fig. 4.19 and it is immediately apparent that the waveform of magnetizing current is far from sinusoidal.

If the hysteresis loop for the transformer is known, it is possible to determine a more accurate form for the waveform of the no-load current, since the component of the no-load current required to set up the flux and to supply the hysteresis loss can be found in a manner similar to that used to determine the magnetizing current from the magnetization curve. Then Table 4.2 can

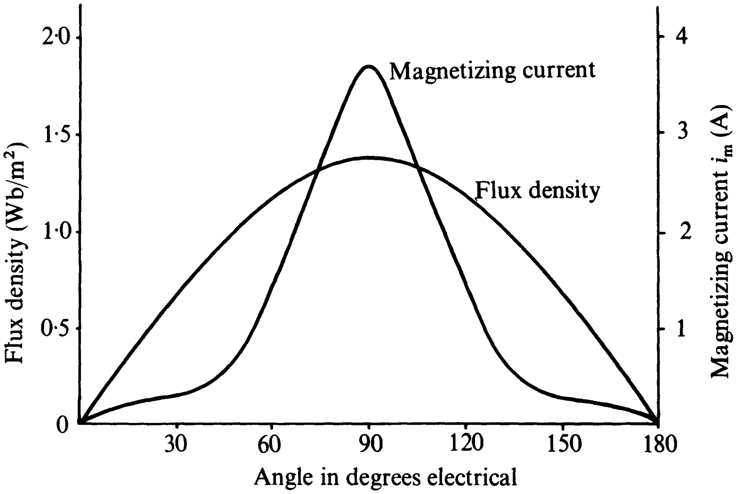


Fig. 4.19. Waveforms of flux density and magnetizing current.

TABLE 4.2

Time (θ°e)	*Flux density* ($b = 1{\cdot}4 \sin \theta$)	*Magnetizing force* (h)	*Current* (i)
0	0	170	0·425
15	0·366	230	0·575
30	0·7	290	0·725
45	0·99	400	1·0
60	1·21	672	1·68
75	1·35	1100	2·75
90	1·4	1500	3·75
105	1·35	852	2·13
120	1·21	332	0·83
135	0·99	104	0·26
150	0·7	−40	−0·1
165	0·366	−104	−0·26
180	0	−172	−0·43

be drawn up in exactly the same manner as Table 4.1 for the hysteresis loop given in Fig. 4.20.

Values of current must be obtained for a range of θ from 0 to 180° and the current obtained by this method is the sum of the magnetizing current i_m and the hysteresis loss component i_h of the core loss component of current. This waveform is shown in Fig. 4.21, and if a sinusoidal component of current to take account of the eddy current component i_e of the core loss component of current is added to this waveform, the no-load current i_0 can be drawn. The

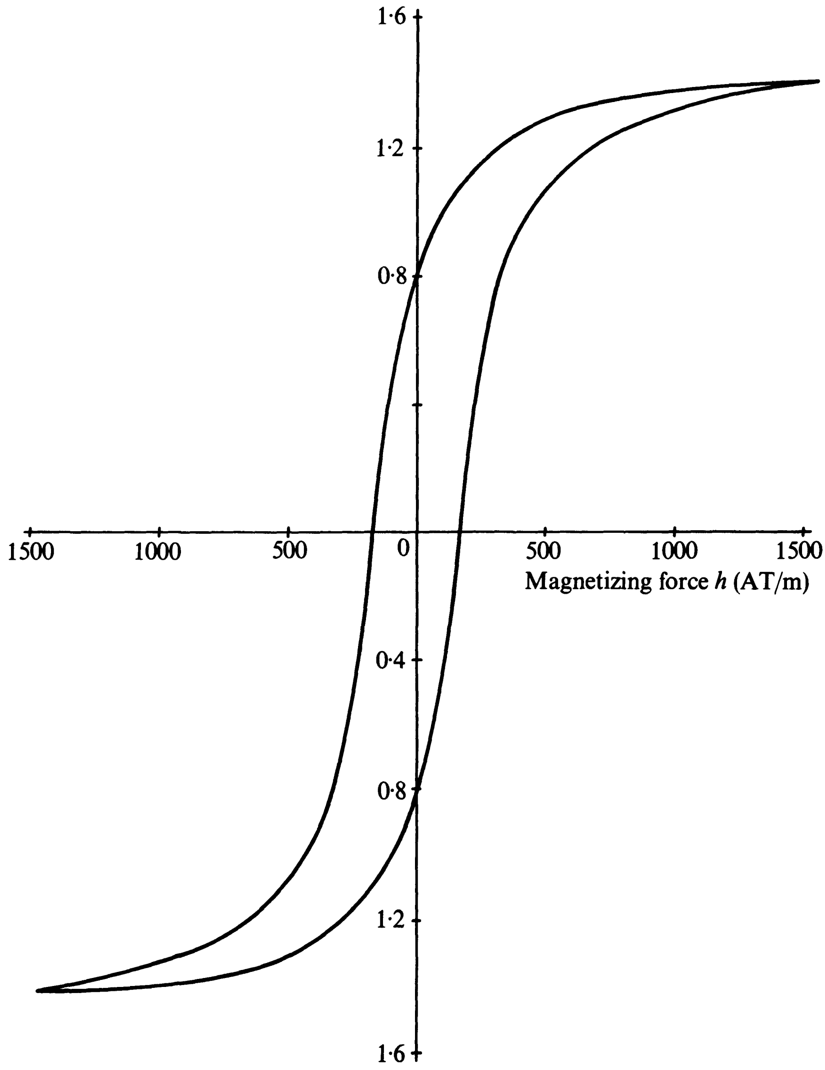


Fig. 4.20. Hysteresis loop.

waveform of the no-load current i_0 obtained by this method is also shown in Fig. 4.21. It can then be said that, to a close degree of accuracy, the no-load current can be subdivided into a sinusoidal waveform of current which provides for the core loss in the transformer, and a non-sinusoidal waveform of magnetizing current. Thus the waveform of the no-load current can be seen to be a periodic function of time of period 360° electrical. As such it can be resolved into a family of sinusoidally varying quantities of different frequencies. An analysis of the waveform shown in Fig. 4.21 gives the result

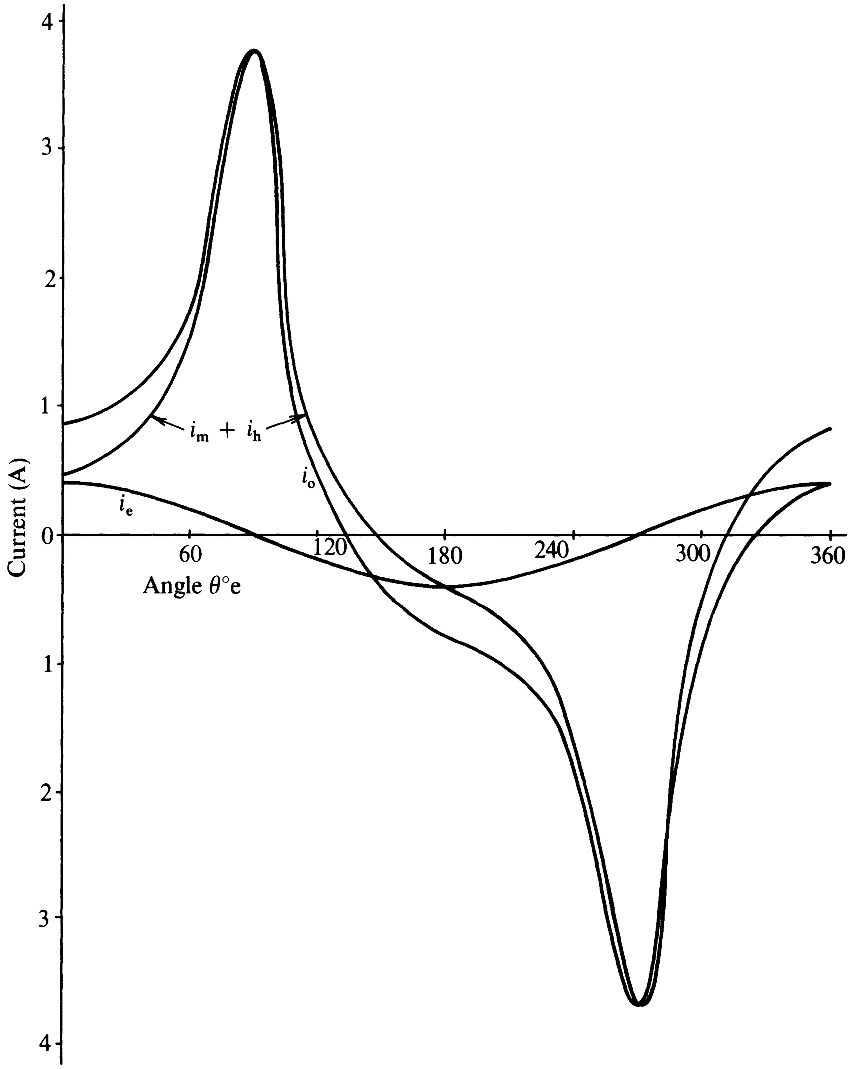


Fig. 4.21. Waveform of no-load current.

No-load current $i_0 = 2{\cdot}05 \sin\theta + 0{\cdot}85 \cos\theta - 1{\cdot}02 \sin 3\theta$

$$+ 0{\cdot}48 \sin 5\theta - 0{\cdot}2 \sin 7\theta + \cdots.$$

Then

$$\text{Core loss component of no-load current, } i_c = 0{\cdot}85 \cos\theta$$

Magnetizing current, $i_m = 2{\cdot}05 \sin\theta - 1{\cdot}024 \sin 3\theta$

$$+ 0{\cdot}48 \sin 5\theta - 0{\cdot}2 \sin 7\theta + \cdots.$$

The third harmonic component of no-load current has an r.m.s. value which is equal to 41% of the r.m.s. value of the total no-load current and the fifth harmonic component of no-load current has an r.m.s. value which is equal to 19% of the r.m.s. value of the total no-load current. Thus in problems directly associated with no-load conditions, the presence of harmonics is of considerable importance. It must be noted, however, that the no-load current should, in practice, never exceed 5% of the rated current for the transformer and that, under normal operating conditions on load, the sinusoidal requirements of the load component of current will predominate the current requirements of the transformer and the harmonics in the waveform of no-load current will be relatively unimportant.

It can then be concluded that if the voltage applied to a transformer is sinusoidal and the mutual flux set up is assumed to be sinusoidal, the no-load current must have a large harmonic content. If, on the other hand, it is assumed that the no-load current of the transformer is sinusoidal, the mutual flux and primary and secondary e.m.f.s must have an harmonic content.

4.10. 3-Phase transformer connections

There are many ways in which three single-phase transformers can be connected to form a 3-phase transformer bank and, in these circumstances, some standard method of terminal marking becomes essential. It is possible to connect both the primary and secondary windings in star (Y), delta (Δ or D), or zigzag (⋎ or Z); in all cases the primary terminals will be designated by capital letters (ABC) and the secondary terminals by small letters (abc). The successive terminals in each phase will be numbered in sequence (A_1, A_2, etc.), and when the instantaneous value of the voltage in a primary phase (say A) is in the direction of A_1 to A_2, the corresponding secondary phase a voltage will always be assumed to be in the direction of a_1 to a_2. The British Standard Specification (No. 171) for polyphase transformer connections lists transformers into four main groups according to the phase shift between corresponding primary and secondary *line* voltages, and each connection within a main group will be specified by the form of the primary and secondary connections. This phase shift is the angle by which the primary line voltage leads the corresponding secondary line voltage and is measured in unit increments of 30° in a clockwise direction. Thus the designation Yd11 specifies a star connected primary, a delta connected secondary and an $11 \times 30° = 330°$ angle between corresponding primary and secondary line terminals with the secondary leading the primary. Table 4.3 summarizes most of the standard connections and shows, under idealized conditions, the voltage and current relationships for balanced primary line voltages V with a primary to secondary turns ratio t. Some of the more important of these connections will be considered further.

(a) Delta-delta connection (Dd0 or Dd6)

The primary and secondary phase windings must both be insulated for the full-line voltages while the phase currents under balanced conditions are each 57·7% of their line values. This connection is useful for relatively large, low voltage transformers but suffers from the disadvantage that no neutral connections are available. One of the main advantages of this connection is the fact that one phase can be removed without disrupting the operating system. This condition, known as the open delta or vee connection is capable of supplying $1/\sqrt{3} = 57{\cdot}7\%$ of the load supplied by the full connection.

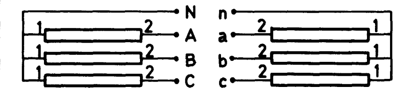
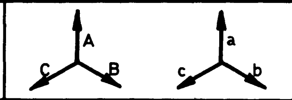
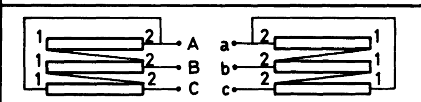
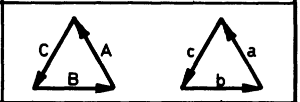
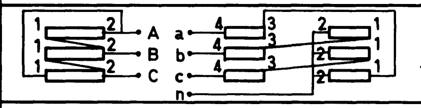
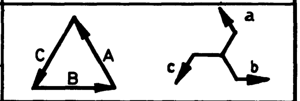
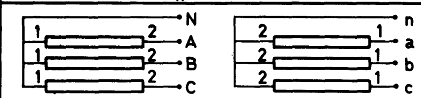
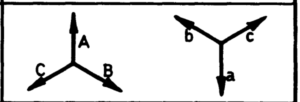
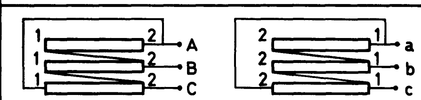
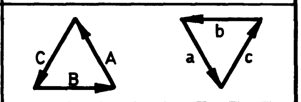
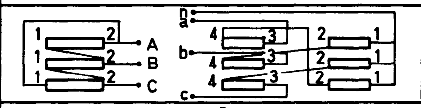
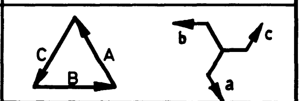
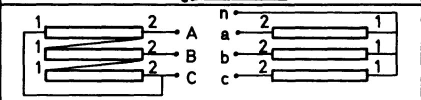
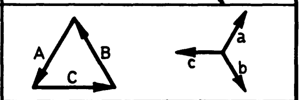
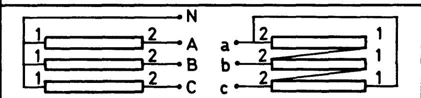
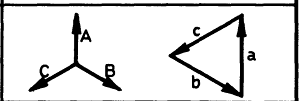
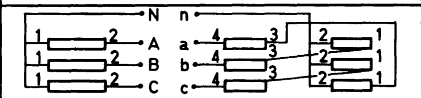
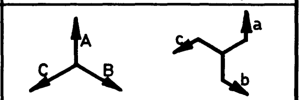
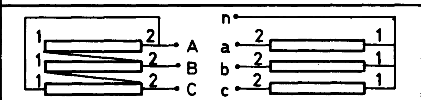
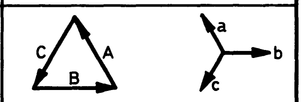
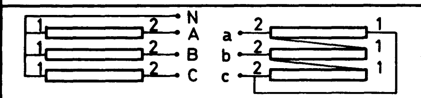
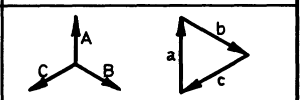
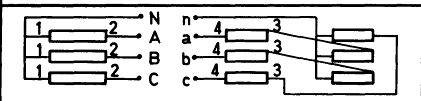
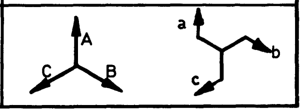
It has been noted previously that, with sinusoidal applied voltages, the presence of a third-harmonic component of no-load current can have an important influence on the operation of 3-phase transformer banks. If the phase currents in a delta-connected circuit each contain a third harmonic it follows that since the fundamentals are 120° out of phase, the third harmonics are in phase. Thus the line currents will contain no third-harmonic component and these third-harmonic currents will circulate around the delta. Thus, any transformer with a delta-connected winding will operate with sinusoidal voltages on both sides with a third-harmonic current circulating within the delta.

(b) Star-star connection (Yy0 or Yy6)

This connection can be used for relatively small high voltage transformers and has the advantage that a neutral connection can be made on both sides of the transformer. If, however, it is necessary to operate without a primary neutral conductor, severe imbalance in the secondary line to neutral loads must be avoided when the bank consists of three single-phase transformers or a 3-phase shell-type transformer.

When three single-phase transformers connected in star-star operate with a primary neutral conductor, the neutral, under balanced loading conditions, will carry only third-harmonic current. Since the third-harmonic component of the no-load current has a return path through the neutral conductor, the phase voltages on each side of the transformer will be sinusoidal. If the primary neutral conductor is removed, third-harmonic currents cannot exist. Thus the phase voltages on both sides of the transformer will contain a

TABLE 4.3

GROUP NUMBER SYMBOL PHASE SHIFT	WINDINGS AND TERMINALS Primary Secondary	PHASOR DIAGRAMS Primary Secondary
1 1 Yy0 0°		
1 2 Dd0 0°		
1 3 Dz0 0°		
2 1 Yy6 180°		
2 2 Dd6 180°		
2 3 Dz6 180°		
3 1 Dy1 -30°		
3 2 Yd1 -30°		
3 3 Yz1 -30°		
4 1 Dy11 +30°		
4 2 Yd11 +30°		
4 3 Yz11 +30°		

pronounced third-harmonic component and the neutral point voltage will oscillate at triple frequency although the line voltages will be sinusoidal. When a secondary neutral conductor is provided, it would be, in many cases, earthed and the resulting third-harmonic currents to earth could cause interference with protection and telephone circuits. In these circumstances, it is usual to provide such transformers with a third set of windings connected in delta. Such a winding is known as a tertiary winding and provides a path for the third-harmonic currents necessary to produce sinusoidal primary and secondary phase voltages. The tertiary winding can also be used for auxiliary loading purposes and, under unbalanced loading conditions, helps to balance the load more evenly between the primary phases. The rating of the tertiary winding will, of course, depend on its loading but will normally be a minimum of 30% of the transformer rating in order to protect this winding under short-circuit conditions.

An alternate method of reducing the effects of third-harmonic fluxes for the star-star connection is to use the three limb core-type construction. At any instant in time, the third-harmonic fluxes in the three separate limbs of the core are in the same direction and their only possible return path is through high reluctance air. Under these conditions the third-harmonic fluxes are considerably reduced and their effect on the phase voltages can normally be neglected. The use of this form of construction also has a very favourable influence on the performance of the transformer under unbalanced loading conditions.

(*c*) *Star-zigzag* (*Yz1 and Yz11*)

The zigzag connection involves the use of two separate identical secondary windings on each phase and, for the same output voltage, requires more copper than the corresponding single winding secondary. However, the secondary phase voltages are obtained by the addition of two voltages displaced by 60° and should each of these two voltages contain a third harmonic, these third harmonics will cancel and not appear in the phase voltages. Under unbalanced loading conditions each secondary phase current will be reflected in two primary phases and this effect leads to a much better performance under these conditions.

The zigzag secondary connection is widely used for three-anode rectifier circuits in which each anode conducts for one-third of a cycle.

(*d*) *Delta-star* (*Dy1 and Dy11*) *and star-delta* (*Yd1 and Yd11*)

These forms of connection can be used to combine the advantages of both star and delta connection into one transformer and are widely used in power distribution systems. The star connection can be used for a stable four-wire supply and there will be no neutral displacement under unbalanced loading conditions.

4.11. Parallel operation of 3-phase transformers

The parallel operation of 3-phase transformers is particularly important in distribution systems and the following conditions must be satisfied if the transformers are to operate satisfactorily in parallel:

(a) their voltage ratios must be the same;

(b) the phase shift between corresponding primary and secondary terminals on each transformer must be the same. It immediately follows that all transformers in the same main group can be connected in parallel. It is, however, possible to connect a transformer with a 330° phase shift (Group 4) in parallel with a transformer with a 30° phase shift (Group 3) if the phase sequence of the primary and secondary of one transformer are reversed.

It has been shown previously that, if two single-phase transformers operating in parallel are to share the total load in proportion to their ratings, the magnitudes of their per-unit impedances must be the same and that, if they are to operate at the same power-factor, the phase angle of their impedances must be the same. The same conclusions can be applied directly to the parallel operation of polyphase transformers.

Any of the equivalent circuits derived previously for the single-phase transformer can be applied directly to any one phase of a 3-phase transformer under balanced loading conditions and it follows that the theory of chapter 4 can be applied on a per-phase basis to the parallel operation of 3-phase transformers. It is, however, preferable to perform calculations on a per-unit basis, particularly in cases where the primary and secondary connections are different.

4.12. The auto-transformer

It has previously been assumed that the transformer is a two-winding device but the windings of a normal transformer can be joined as shown in Fig. 4.22(a) without affecting the performance. Such an arrangement is known as

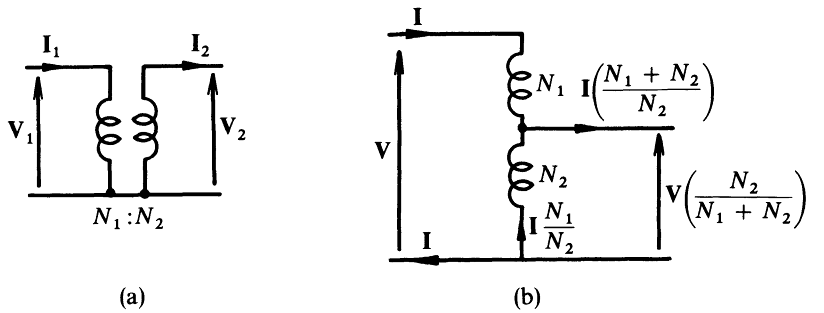


Fig. 4.22. The auto-connection.

an auto-transformer and is normally represented by the arrangement shown in Fig. 4.22(b).

The amount of copper used for an auto-transformer will be less than that for a two winding transformer and the reduction in the effective core length will result in reduced iron losses. It follows that both the capital and running cost of an auto arrangement will be less than that of a two-winding winding. The direct electrical connection between the high- and low-voltage sides is, however, one disadvantage of the auto-connection.

Tutorial Problems

1. A single-phase core-type transformer is to be designed to have a primary voltage of 33 kV and a secondary voltage of 6·6 kV. If the maximum flux density permissible is 1·2 Wb/m^2 and the number of primary turns is 1250, calculate the number of secondary turns and the cross-sectional area of the core when the operating frequency is 50 Hz.

   (*Answer:* 250; 0·0993 m^2)

2. The parameters of a 200/100 V, 50 Hz, single-phase transformer are

$$R_1 = 0{\cdot}1\ \Omega; \quad X_1 = 0{\cdot}4\ \Omega; \quad R_2 = 0{\cdot}03\ \Omega; \quad X_2 = 0{\cdot}1\ \Omega$$

$$g = 1{\cdot}2 \times 10^{-4}\ \text{mho},$$

$$b = 4 \times 10^{-4}\ \text{mho}.$$

   The transformer supplies a secondary load current of 20 A at power factor 0·8 lag with rated secondary voltage. Draw a complete phasor diagram to scale and determine the primary applied voltage and efficiency under these conditions.

   (*Answer:* 216·5 V; 97·1%)

3. A 200/400 V single-phase transformer gave the following test results:
   (a) with the *low*-voltage winding short-circuited, measurements taken on the high-voltage side were: 20 V, 10 A, 100 W.
   (b) with the *high*-voltage winding open-circuited, measurements taken on the low-voltage side were: 200 V, 1 A, 60 W.

   Determine the circuit constants and draw the approximate equivalent circuit as seen from the low-voltage side for a rating of 4 kVA.

   If a load impedance $(80 + j35)\ \Omega$ is connected to the high-voltage terminals, determine the percentage regulation and efficiency when 200 V is applied to the low-voltage terminals.

   (*Answer:* $R = 0{\cdot}25$; $X = 0{\cdot}433$; $g = 1{\cdot}5 \times 10^{-3}$; $b = 4{\cdot}77 \times 10^{-3}$; 2%; 95·5%)

4. The results of the open-circuit and short-circuit tests on a single-phase transformer are as follows:

	Voltage p.u.	*Current p.u.*	*Power p.u.*
Open-circuit	1·0	0·025	0·01
Short-circuit	0·04	1·0	0·012

   Derive the per-unit values of its parameters and find the power-factor at which the voltage regulation is zero and the corresponding efficiency.

   (*Answer:* $R = 0{\cdot}012$; $X = 0{\cdot}0382$; $g = 0{\cdot}01$; $b = 0{\cdot}0229$; 0·947 lead, 97·7%)

5. A 200/100 V, 50 Hz, single-phase transformer has a primary leakage impedance of $0{\cdot}15 + j0{\cdot}76\ \Omega$ and an actual secondary leakage impedance of $0{\cdot}04 + j0{\cdot}19\ \Omega$. The transformer supplies a load of impedance $3{\cdot}92 + j2{\cdot}62\ \Omega$. Calculate the secondary terminal voltage, the input current and power-factor, the regulation and the efficiency of the transformer if the primary is supplied at rated voltage.
(*Answer:* 94·4 V; 10 A; 0·8 lag; 5·6%; 97·9%)

6. Show that the voltage regulation of the transformer is given by $[V_{sc}\cos(\phi - \phi_{sc})]/V_{rated}$ where V_{sc} is the voltage required to circulate rated current on short-circuit, $\cos\phi_{sc}$ is the power-factor on short-circuit, and $\cos\phi$ is the operating power-factor.

7. Two single-phase transformers A and B with identical turns ratios and ratings operate in parallel. The per-unit impedance of A is $0{\cdot}006 + j0{\cdot}04$ and that of B is $0{\cdot}009 + j0{\cdot}05$. If A operates on full load at power-factor 0·8 lag, calculate the per-unit load and operating power-factor of B.
(*Answer:* 0·801, 0·825 lag)

8. A 400 kVA, 33/3·3 kV, single-phase transformer is connected in parallel with a 500 kVA, 33/3·3 kV, single-phase transformer and the combination supplies a total load of 800 kVA at power-factor 0·8 lagging. The resistances are 1% and 2% and the leakage reactances are 7% and 5% respectively. Calculate the terminal voltage, the kVA output, the current and the power-factor of each transformer.
(*Answer:* 30·1 kV; 304 kVA; 498 kVA; 101 A; 166 A; 0·7; 0·852)

9. The hysteresis loop for the iron core of a 50 Hz, single-phase transformer is as follows:

Flux density b (Wb/m^2)	0	0·2	0·4	0·6	0·8	1·0	1·15
Magnetizing force h (AT/m)	110	145	200	240	390	555	800
	−110	−80	−30	0	155	370	800

Draw one cycle of the waveform of no-load current for this transformer when the applied voltage is sinusoidal and one cycle of the waveform of voltage when the no-load current is sinusoidal.

10. The total iron loss for a 440 V, 50 Hz, transformer is 2500 W. When the applied voltage is 220 V at 25 Hz the corresponding loss is 850 W. Calculate the eddy current loss and the hysteresis loss at normal voltage and frequency.
(*Answer:* 1600 W; 900 W)

11. The total iron loss at constant flux density in a transformer varies with frequency as follows:

Total iron loss (W)	46	63	78	100	124	175	230
Frequency (Hz)	25	33·3	40	50	60	80	100

Determine the hysteresis and eddy current loss at 50 Hz.
(*Answer:* 84·5 W; 15·5 W)

12. The input current to a 3-phase, step-down transformer connected to an 11 kV supply system is 14 A. Calculate the secondary line voltage and current for (a) star-star, (b) delta-star, and (c) star-delta connection if the phase turns ratio is 44.
(*Answer:* 250 V; 616 A; 432 V; 355 A; 144 V; 1070 A)

13. An 11,000/415 V 3-phase transformer with high voltage tappings of $\pm 2\frac{1}{2}\%$ is designed to operate on 4 V per turn. Find the number of turns required on each winding for (a) star-star, (b) delta-star, and (c) delta-zigzag connection.

(*Answer:* 1630/62; 2818/62; 2818/69)

14. When phasing-out two 3-phase 11,000/400 V delta-star transformers with their neutrals joined and their h.v. terminals connected to the h.v. supply the following voltmeter readings were obtained:

$$a_2 - a_2,\ 460\ \text{V}; \qquad b_2 - b_2,\ 230\ \text{V}; \qquad c_2 - c_2,\ 230\ \text{V}.$$

Deduce how the incoming transformer differs from the original transformer and explain how the incoming transformer can be made to operate in parallel with the original transformer.

15. A delta-zigzag, 3-phase transformer is required to operate in parallel with a star-star, 3-phase transformer with a turns ratio $N = 3$. Find the required phase turns ratio for the delta-zigzag transformer.

(*Answer:* 4·5)

5. Polyphase induction motors

5.1. Derivation of equivalent circuits

It has been shown in chapter 1 that the polyphase induction motor can be considered as a polyphase transformer with variable frequency in the short-circuited secondary winding and an equivalent circuit can be derived per phase of the motor in the same general way as that of a transformer. It is apparent that each winding will have resistance and leakage inductance. The leakage reactance of the rotor will, however, be dependent on the frequency of the rotor current and, since the rotor frequency is Sf, the slip frequency, where $S = (N_S - N)/N_S$ is the slip and f is the supply frequency, the rotor leakage reactance is given by SX_2 where X_2 is the standstill value of the rotor leakage reactance. Thus an equivalent circuit for the rotor can be drawn as shown in Fig. 5.1. The stator and rotor will, however, have different turns and

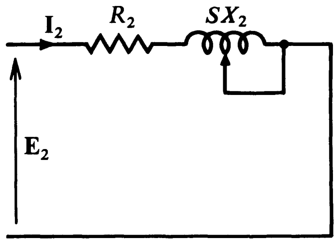


Fig. 5.1. Rotor equivalent circuit.

the actual rotor must be made into an equivalent rotor having the same turns as the stator. At a corresponding flux and slip the relation between the actual rotor induced voltage $\mathbf{E}_2$ and the equivalent rotor induced voltage $\mathbf{E}_{2_1}$ is

$$\mathbf{E}_{2_1} = a\mathbf{E}_2 \tag{5.1}$$

where a is the ratio, effective stator turns per phase, N_1, to effective rotor turns per phase, N_2.

If the rotors are to be magnetically identical it follows that their m.m.f.s must be equal and the relation between the actual rotor current I_2 and the equivalent rotor current I_{2_1} is

$$I_{2_1} = \frac{I_2}{a}. \quad (5.2)$$

Thus the relation between the slip-frequency leakage impedance $\mathbf{Z}_2 = R_2 + jSX_2$ of the actual rotor and the slip-frequency leakage impedance $\mathbf{Z}_{2_1} = R_{2_1} + jSX_{2_1}$ of the equivalent rotor is

$$\mathbf{Z}_{2_1} = R_{2_1} + jSX_{2_1} = \frac{\mathbf{E}_{2_1}}{\mathbf{I}_{2_1}} = a^2\frac{\mathbf{E}_2}{\mathbf{I}_2} = a^2(R_2 + jSX_2) = a^2\mathbf{Z}_2. \quad (5.3)$$

The equivalent circuit for a magnetically equivalent rotor is shown in Fig. 5.2, and the voltages, currents, and impedances in this circuit are now referred to the stator turns. It will be assumed that the referred rotor constants are known quantities.

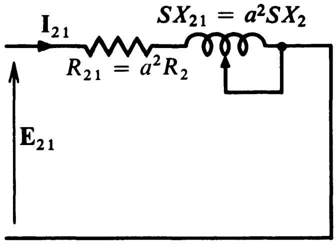


Fig. 5.2. Rotor equivalent circuit referred to stator turns.

Now the resultant flux in the machine can be assumed to link both stator and rotor and by the laws of electromagnetic induction this flux induces a slip frequency e.m.f. $\mathbf{E}_2$ in the rotor and a mains frequency e.m.f. $\mathbf{E}_1$ in the stator. The relative speed of the resultant flux with respect to the rotor is the slip S times its relative speed with respect to the stator and it follows that, if the resultant flux is assumed constant,

$$E_1 = 4{\cdot}44k_wN_1f\Phi, \qquad E_2 = 4{\cdot}44k_wN_2(Sf)\Phi.$$

Then

$$\frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{N_2S}{N_1}, \quad \text{or} \quad \frac{\mathbf{E}_2N_1}{N_2} = \mathbf{E}_{2_1} = S\mathbf{E}_1. \quad (5.4)$$

It follows that, since, from (5.3) $\mathbf{E}_{2_1}/\mathbf{I}_{2_1} = (R_{2_1} + jSX_{2_1})$

$$\mathbf{E}_{2_1} = S\mathbf{E}_1 = \mathbf{I}_{2_1}(R_{2_1} + jSX_{2_1})$$

or

$$\mathbf{E}_1 = \mathbf{I}_{2_1}\left(\frac{R_{2_1}}{S} + jX_{2_1}\right). \quad (5.5)$$

The equivalent circuit for each phase of the stator winding of the machine can be derived in exactly the same way as that for a transformer and is shown in Fig. 5.3.

It should be noted that the input current I_1 can be resolved into two components, the load current I_{2_1} and the no-load current I_0. The m.m.f. of the load current balances the m.m.f. of the rotor current. The equivalent circuits for the stator and for the rotor when referred to the stator turns can then be combined as shown in Fig. 5.4.

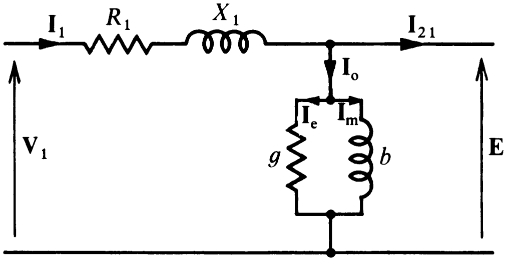


Fig. 5.3. Stator equivalent circuit.

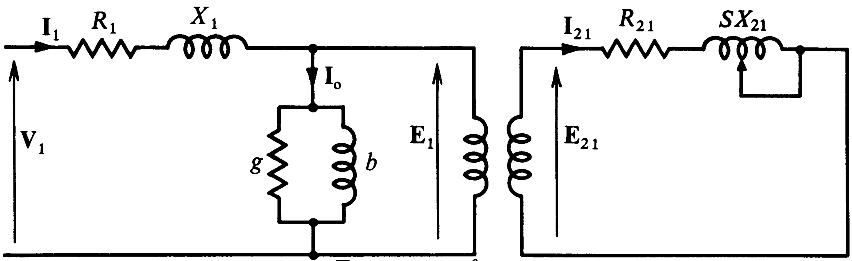


Fig. 5.4. Combined equivalent circuit.

It can be seen from (5.5) that the circuit shown in Fig. 5.4 can be reduced to the usual exact equivalent circuit per-phase of a polyphase induction motor shown in Fig. 5.5, in which the resistance R_{2_1}/S is an effective resistance

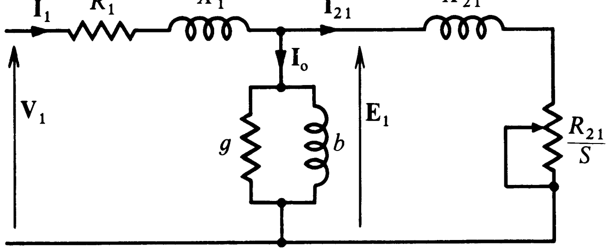


Fig. 5.5. Exact equivalent circuit.

combining the effects of shaft load and actual rotor resistance. If, as is usual, it is assumed that the iron losses in the machine are constant, the resistive branch in the no-load circuit can be removed and the equivalent circuit redrawn as shown in Fig. 5.6. It should be noted that all quantities used in the equivalent circuit must be expressed as phase quantities.

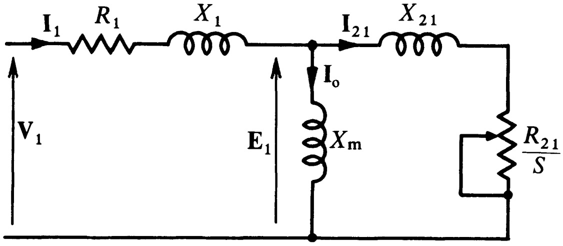


Fig. 5.6. Final form of exact equivalent circuit.

5.2. Power balance equations

For an m phase machine with an equivalent circuit of the form shown in Fig. 5.6,

$$\text{Input power} = mV_1I_1 \cos \phi_1,$$

where ϕ_1 is the angle between V_1 and I_1.

$$\text{Stator copper loss} = mI_1{}^2R_1. \tag{5.6}$$

$$\text{Power transferred to rotor} = mI_{2_1}^2 \frac{R_{2_1}}{S}. \tag{5.7}$$

$$\text{Rotor copper loss } P_C = mI_{2_1}^2 R_{2_1}. \tag{5.8}$$

$$\text{Gross output power } P_g = (5.7) - (5.8) = mI_{2_1}^2 R_{2_1} \frac{1-S}{S}. \tag{5.9}$$

Fixed losses are friction + windage + iron losses.
Nett output power = gross output power − fixed losses.

Now $$\frac{2\pi N}{60} T_g = P_g = mI_{2_1}^2 (R_{2_1}) \left(\frac{1-S}{S}\right).$$

But $$N = N_S(1 - S).$$

Then $$\text{Gross torque } T_g = \frac{mI_{2_1}^2 \dfrac{R_{2_1}}{S}}{\dfrac{2\pi}{60} N_S} \text{ N.m.} \tag{5.10}$$

Now $2\pi N_S/60$ is a constant for a particular machine and it follows that

$[(2\pi/60)N_S]T_g$, which is equal to the power transferred across the air-gap, is a measure of the gross torque.

Thus the power transferred across the air-gap is said to be the gross torque in synchronous watts.

Then
$$\text{Gross torque } T_g = mI_{2_1}^2 \frac{R_{2_1}}{S} \text{ syn. watts.} \tag{5.11}$$

But rotor copper loss $P_C = mI_{2_1}^2 R_{2_1}$ watts, and gross output power $P_g = mI_{2_1}^2 R_{2_1}[(1 - S)/S]$ watts. Then

$$\text{Torque (in syn. watts)} = \frac{\text{rotor copper loss}}{S}$$

$$= \frac{\text{gross output power}}{(1 - S)}. \tag{5.12}$$

The whole performance of an induction motor can be calculated on the basis of equivalent circuits but some simplifications can be made to the circuit by the application of Thevenin's theorem and the Maximum Power Transfer Theorem* if torque slip or power-slip relationships are required.

5.3. Torque calculations

Consider the exact equivalent circuit shown in Fig. 5.7(a). The circuit viewed from the points 'a' and 'b' can be replaced by a single equivalent voltage $\mathbf{V}_e$ and a single equivalent impedance $\mathbf{Z}_e = R_e + jX_e$ such that

$$\mathbf{V}_e = \mathbf{V}_1 \frac{jX_m}{R_1 + j(X_1 + X_m)} \tag{5.13}$$

$$\mathbf{Z}_e = \frac{(R_1 + jX_1)jX_m}{R_1 + j(X_1 + X_m)}. \tag{5.14}$$

In general, it can be assumed that $X_1 + X_m$ is much greater than R_1 and these equations then reduce to

$$\mathbf{V}_e = \mathbf{V}_1 \frac{X_m}{X_1 + X_m} \tag{5.15}$$

$$\mathbf{Z}_e = R_e + jX_e = \frac{(R_1 + jX_1)X_m}{X_1 + X_m}. \tag{5.16}$$

* Thevenin's theorem states that any network of circuit elements and voltage sources as viewed from two terminals can be replaced by a single source in series with a single impedance. The source voltage is the voltage that would appear across those terminals when they are open-circuited and the impedance is that viewed from those terminals when all voltage sources in the original network are short-circuited.

The Maximum Power Transfer Theorem states that maximum power will be transferred from a source of inductive impedance Z_1 to a sink of inductive impedance Z_2 when $Z_1 = Z_2$.

Thus the exact equivalent circuit can be redrawn as shown in Fig. 5.7(b). Now from (5.11),

$$\text{Torque } T_e = mI_{2_1}^2 \frac{R_{2_1}}{S}.$$

Thus

$$T_e = \frac{mV_e{}^2 R_{2_1}/S}{(R_e + R_{2_1}/S)^2 + X^2} \text{ syn. watts.} \tag{5.17}$$

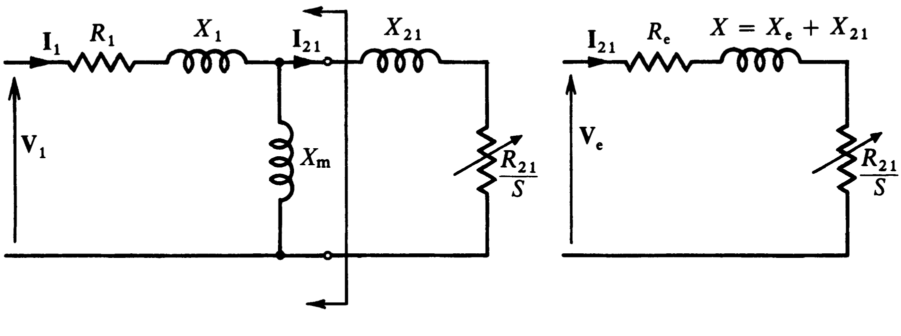


Fig. 5.7. Derivation of Thevenin equivalent circuit: (a) Usual circuit. (b) Thevenin equivalent.

The general shape of the torque–slip curve for an induction machine is shown in Fig. 5.8 and the machine will operate as a motor when $1 \geqslant S > 0$, as a generator when $S < 0$ and as a brake when $S > 1$. The machine must be

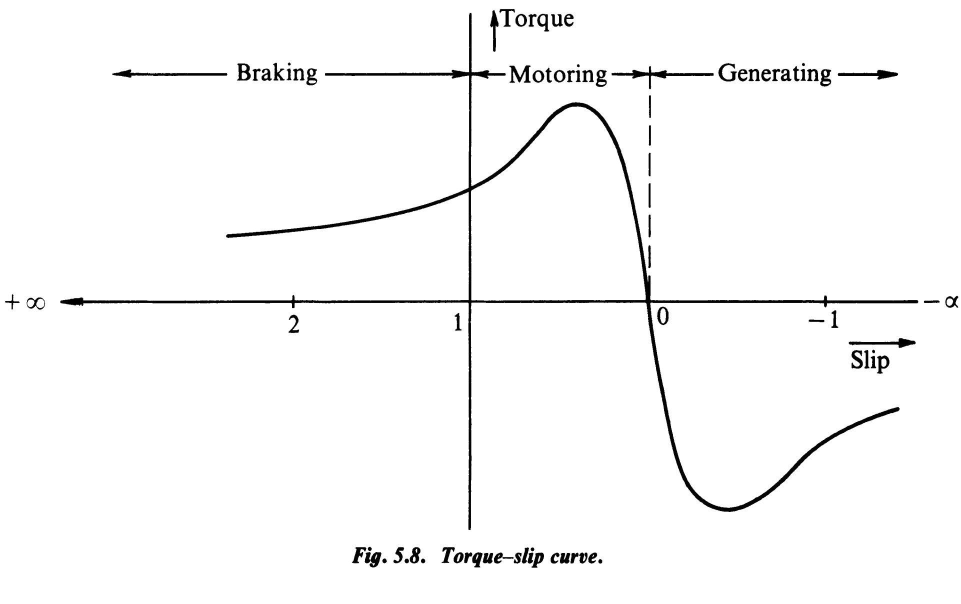


Fig. 5.8. Torque–slip curve.

driven above synchronous speed for generator operation and must be driven against the direction of rotation of its m.m.f. for operation as a brake.

Since the torque for synchronous watts is the power dissipated in the resistor R_{2_1}/S, it follows that the torque will be a maximum when the power dissipated in R_{2_1}/S is a maximum. Then, by the Maximum Power Transfer Theorem, maximum torque will occur when $|R_e + jX| = R_{2_1}/S$. Thus the slip S_t for maximum torque is given by

$$S_t = \frac{R_{2_1}}{(R_e^2 + X^2)^{\frac{1}{2}}}. \tag{5.18}$$

When this value of slip is substituted in the general equation for torque, (5.17), the value of maximum torque T_m is given by

$$T_m = \frac{mV_e^2\sqrt{(R_e^2 + X^2)}}{[R_e + \sqrt{(R_e^2 + X^2)}]^2 + X^2}.$$

On simplification

$$T_m = \frac{mV_e^2}{2[R_e + \sqrt{(R_e^2 + X^2)}]}. \tag{5.19}$$

It should be noted that from (5.18), the slip S_t at which maximum torque is available is a direct function of the rotor resistance but that, from (5.19), the value of maximum torque is independent of the rotor resistance. The typical torque–slip curves for a motor with variable rotor resistance are shown in Fig. 5.9 and it can be seen that the speed of an induction motor can be controlled by variation of the rotor resistance. Since the starting torque ($S = 1$) and current of the motor also depend on the value of rotor resistance, variation of rotor resistance can also be used as a starting method. Both of these techniques can, of course, only be applied to wound-rotor induction motors.

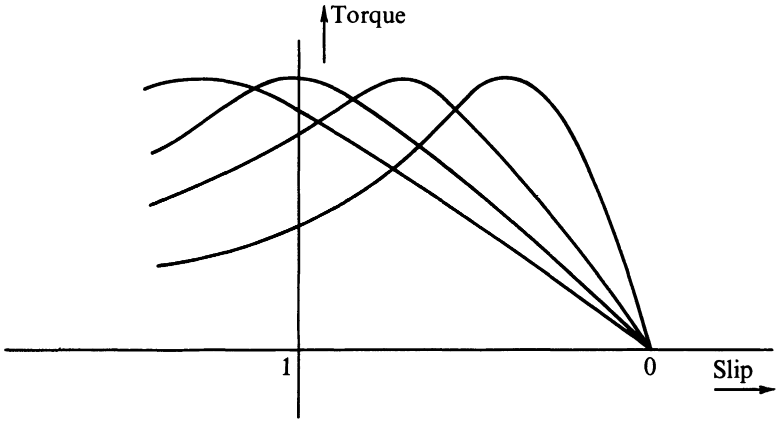


Fig. 5.9. Torque–slip curves for variable rotor resistance.

It is sometimes convenient to express the torque in terms of dimensionless parameters (ref. 1) and a torque ratio t can be defined as the ratio of the torque T at any slip S to the maximum torque T_m at slip S_t; that is,

$$t = \frac{T}{T_m} = \frac{2[R_e + \sqrt{(R_e^2 + X^2)}]R_{2_1}/S}{(R_e + R_{2_1}/S)^2 + X^2}.$$

The slip S_t for maximum torque is given by $S_t = R_{2_1}/\sqrt{(R_e^2 + X^2)}$, so that

$$R_{2_1} = S_t\sqrt{(R_e^2 + X^2)}, \qquad \text{or} \quad R_{2_1} = S_t R_e \sqrt{\{1 + (X/R_e)^2\}}.$$

Then
$$t = \frac{2R_e\left[1 + \sqrt{\left\{1 + \left(\frac{X}{R_e}\right)^2\right\}}\right]\frac{S_t}{S}R_e\sqrt{\left\{1 + \left(\frac{X}{R_e}\right)^2\right\}}}{R_e^2\left\{\left[1 + \frac{S_t}{S}\sqrt{\left\{1 + \left(\frac{X}{R_e}\right)^2\right\}}\right]^2 + \left(\frac{X}{R_e}\right)^2\right\}}. \tag{5.20}$$

If X/R_e is defined as a quality factor Q the torque ratio (5.20) can be written as

$$t = \frac{2\frac{S_t}{S}\{1 + \sqrt{(1 + Q^2)}\}\sqrt{(1 + Q^2)}}{1 + 2\frac{S_t}{S}\sqrt{(1 + Q^2)} + \left(\frac{S_t}{S}\right)^2(1 + Q^2) + Q^2}$$

or
$$t = \frac{1 + \sqrt{(1 + Q^2)}}{1 + \frac{1}{2}\left(\frac{S}{S_t} + \frac{S_t}{S}\right)\sqrt{(1 + Q^2)}}. \tag{5.21}$$

If the stator resistance can be neglected, this equation reduces to

$$t = \frac{2}{\frac{S}{S_t} + \frac{S_t}{S}}. \tag{5.22}$$

5.4. Output power calculations

The gross output power (P_g) is given by (5.9) as

$$P_g = mI_{2_1}^2 R_{2_1}\frac{1 - S}{S} \text{ watts}$$

or
$$P_g = \frac{mV_e^2 R_{2_1}\frac{1 - S}{S}}{\left(R + R_{2_1}\frac{1 - S}{S}\right)^2 + X^2} \text{ watts} \tag{5.23}$$

where $R = R_e + R_{2_1}$ and $X = X_e + X_{2_1}$.

Again, by the use of the Maximum Power Transfer Theorem, it follows that the gross output power will be a maximum when $|R + jX| = R_{2_1}[(1 - S)/S]$. Thus the slip S_p for maximum gross output power is given by

$$S_p = \frac{R_{2_1}}{R_{2_1} + \sqrt{(R^2 + X^2)}}. \qquad (5.24)$$

When this value of slip is substituted in the general equation for gross output power, (5.23), maximum gross output power $P_{g\,max}$ is given by

$$P_{g\,max} = \frac{mV_e^2}{2\{R + \sqrt{(R^2 + X^2)}\}} \text{ watts.} \qquad (5.25)$$

It should be noted, at this stage, that the resistance $R = R_1 + R_{2_1}$ is a function of rotor resistance and that the value of maximum gross output power is not independent of the rotor resistance.

Comparison of (5.18) and (5.24) shows that the slip S_t for maximum torque is not the same as the slip S_p for maximum gross output power.

EXAMPLE 5.1. A 37·3 kW, 3-phase, 4-pole, 50 Hz, induction motor has a full-load efficiency of 85%. The friction and windage losses are one-third of the no-load losses and the rotor copper loss equals the iron loss at full load. Find the full-load speed. Stator resistance can be neglected.

Solution: Now efficiency = 0·85 = nett output/(nett output + losses).
Nett output = 37 300 W.
Losses = copper loss P_c + iron loss P_i + friction loss P_f.
No-load loss = $P_i + P_f = 3\,P_f$. $\therefore\ P_f = \frac{1}{2}P_i$.
On full load $P_c = P_i$. Losses = $P_c + P_c + \frac{1}{2}P_c = \frac{5}{2}P_c$.

Then $$0{\cdot}85 = \frac{37\,300}{37 \times 300 + 2{\cdot}5P_c}, \quad \text{i.e., } P_c = 2640 \text{ W.}$$

Now, from the equivalent circuit,

Gross output power $$P_g = mI_{2_1}^2 R_{2_1}\frac{1 - S}{S} \text{ W.}$$

Rotor copper loss $$P_c = mI_{2_1}^2 R_{2_1} \text{ W.}$$

Then $$P_c\frac{1 - S}{S} = P_g.$$

Thus $$2640\frac{1 - S}{S} = 37 \times 300 + 2640 + \tfrac{1}{2}(2640)$$

whence, $$S = \frac{1}{16{\cdot}65}.$$

Synchronous speed $N_s = 60f/P = 1500$ r.p.m.
Full-load speed $N = N_s(1 - S) = 1500[1 - (1/16{\cdot}65)] = 1410$ r.p.m.

EXAMPLE 5.2. A 415 V, 3-phase, 6-pole, 50 Hz, star-connected wound rotor, induction motor has the following parameters in ohms per phase.

$$R_1 = 0{\cdot}04; \quad X_1 = 0{\cdot}15; \quad R_{2_1} = 0{\cdot}05; \quad X_{2_1} = 0{\cdot}15; \quad X_m = 9{\cdot}85.$$

Stator: rotor turns ratio per phase is 1·5. Determine
(a) the stator current and the gross torque in N.m when the slip is 0·05,
(b) the maximum gross torque, the slip at which it occurs and the gross output power under these conditions,
(c) the values of external resistance to be inserted to produce 80% of maximum torque at standstill and state which value would be used.

Solution (*a*): The input impedance **Z** is given by

$$\mathbf{Z} = 0{\cdot}04 + j0{\cdot}15 + \frac{(1 + j0{\cdot}15)j9{\cdot}85}{1 + j10}$$

$$= 1 + j0{\cdot}395 = 1{\cdot}075\underline{/21^\circ}\,33'.$$

$$\text{Stator current } \mathbf{I}_1 = \frac{415}{\sqrt{3} \times 1{\cdot}075\underline{/21^\circ\,33'}} = 222\overline{\backslash 21^\circ\,33'}\text{ A.}$$

Input power $= \sqrt{3}\, V_1 I_1 \cos\phi_1 = \sqrt{3} \times 415 \times 222 \cos 21^\circ\,33' = 148{\cdot}2$ kW.

Stator copper loss $= 3{I_1}^2 R_1 = 3 \times 222^2 \times 0{\cdot}04 = 5{\cdot}92$ kW.

Power transferred to rotor = torque in syn. watts

$$= (148{\cdot}2 - 5{\cdot}92)10^3 = 142{\cdot}28 \times 10^3.$$

$$\text{Synchronous speed } N_s = \frac{60 \times 50}{3} = 1000 \text{ r.p.m.}$$

From (5.10), $$\text{Torque in N.m} = \frac{142{\cdot}28 \times 10^3}{\dfrac{2\pi}{60} \times 1000} = 1360 \text{ N.m.}$$

Solution (*b*): Take Thevenin equivalent of stator and magnetizing impedances.

$$R_e = 0{\cdot}04\frac{9{\cdot}85}{10} = 0{\cdot}0394, \qquad X_e = 0{\cdot}15 \times \frac{9{\cdot}85}{10} = 0{\cdot}148.$$

Then torque will be a maximum when

$$|0{\cdot}0394 + j(0{\cdot}148 + 0{\cdot}15)| = \frac{0{\cdot}05}{S}.$$

Slip S_t for maximum torque is given by

$$S_t = \frac{0{\cdot}05}{(0{\cdot}0394^2 + 0{\cdot}298^2)^{\frac{1}{2}}} = 0{\cdot}166.$$

From (5.19),

$$\text{Maximum torque } T_m = \frac{3 \times \left(0{\cdot}985 \times \dfrac{415}{\sqrt{3}}\right)^2}{2[0{\cdot}0394 + (0{\cdot}0394^2 + 0{\cdot}298^2)^{\frac{1}{2}}]}$$

$$= 24{,}500 \text{ syn. watts.}$$

From (5.12),

$$\text{Gross output power} = \text{torque (syn. watts)} \times (1 - S)$$

$$= 24{,}500 \times (1 - 0{\cdot}133) = 21{\cdot}2 \text{ kW}.$$

Solution (*c*)*:* Let external resistance referred to stator turns be R. Then, from (5.17) with $S = 1$,

$$\text{Starting torque } T_s = \frac{3(0{\cdot}985V_1)^2(0{\cdot}05 + R)}{(0{\cdot}0394 + 0{\cdot}05 + R)^2 + 0{\cdot}298^2}.$$

But $T_s = 0{\cdot}8T_m$. Therefore

$$0{\cdot}8\frac{3(0{\cdot}985V_1)^2}{2[0{\cdot}0394 + (0{\cdot}0394^2 + 0{\cdot}298^2)^{\frac{1}{2}}]} = \frac{3(0{\cdot}985V_1)^2(0{\cdot}05 + R)}{(0{\cdot}0394 + 0{\cdot}05 + R)^2 + 0{\cdot}298^2}.$$

On solution, $R = 0{\cdot}481$ or $0{\cdot}191$. But turns ratio is 1·5. Actual value of external rotor resistance = 0·214 or 0·0849. The smaller value of resistance $R = 0{\cdot}0849\ \Omega$ would be used and this is apparent from the torque–slip curves shown in Fig. 5.9.

5.5. Determination of circuit parameters

The parameters of the equivalent circuit of a polyphase induction motor can be obtained from design data but can easily be measured by means of three simple tests, a running light or open-circuit test, a locked rotor or short-circuit test, and a measurement of the d.c. resistance per phase of the stator.

(*a*) *Running light test*

The machine is run up to its no-load speed, and the variation of the input power P_{oc} and the input current I_{oc} for a range of voltages V_{oc} from 125% of rated voltage at rated frequency to the minimum possible to maintain the no-load speed constant are recorded.

The no-load losses P_f are usually assumed to be constant and equal to the value at rated frequency and voltage. Then, at rated voltage,

$$P_f = P_{oc} - mI_{oc}^2R_1 \tag{5.26}$$

where R_1 is the d.c. resistance per phase of the stator winding and m is the number of phases.

Since the slip S under no-load conditions is very small, it is assumed that, during the running light test, the rotor is on open-circuit and the equivalent circuit can be drawn as shown in Fig. 5.10. Then

$$X_{oc} = \sqrt{\left[\left(\frac{V_{oc}}{I_{oc}}\right)^2 - \left(\frac{P_{oc}}{mI_{oc}^2}\right)^2\right]} = X_1 + X_m. \tag{5.27}$$

Thus the running light test will give the sum of the stator leakage reactance and the magnetizing reactance and a value for the fixed losses for the machine.

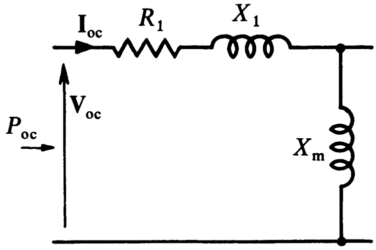


Fig. 5.10. Equivalent circuit for running light test.

(*b*) *Locked rotor test*

The machine rotor is locked so that the motor cannot rotate and the variation of input power P_{sc}, and terminal voltage V_{sc} for a range of input currents at rated frequency up to 125% of rated current are recorded. It should be noted that the value of the input current for a particular applied voltage can, under locked rotor conditions, be affected by the position of the rotor. This effect, however, will be, in general, small for squirrel-cage machines.

The equivalent circuit for the machine under these conditions is exactly that for a transformer with a short-circuited secondary and is shown in Fig. 5.11. However, the applied voltage required to produce rated current

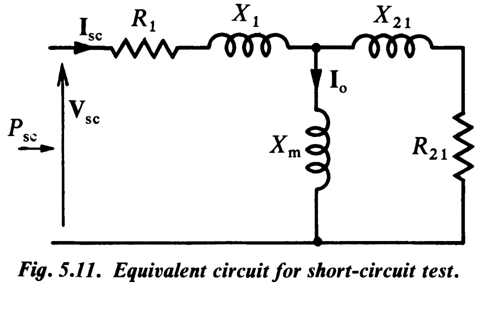


Fig. 5.11. Equivalent circuit for short-circuit test.

with the machine at standstill will only be a small proportion of rated voltage and it is permissible under these conditions to neglect the no-load current. Then, at rated current,

$$P_{sc} = mI_{sc}^2(R_1 + R_{2_1}) \tag{5.28}$$

$$(X_1 + X_{2_1})^2 = \left(\frac{V_{sc}}{I_{sc}}\right)^2 - \left(\frac{P_{sc}}{mI_{sc}^2}\right)^2. \tag{5.29}$$

The sum of the reactances $X_1 + X_{2_1}$ must now be subdivided into its stator and rotor components and the recommended distribution (ref. 2) is shown in Table 5.1.

TABLE 5.1
Fraction of $X_1 + X_{2_1}$

	X_1	X_{2_1}
Class A, normal starting torque and current	0·5	0·5
Class B, normal starting torque, low starting current	0·4	0·6
Class C, high starting torque, low starting current	0·3	0·7
Class D, high starting torque, high running slip	0·5	0·5
Wound rotor	0·5	0·5

Thus values for the stator resistance and leakage reactance, the magnetizing reactance, the standstill values of rotor resistance and leakage reactance and a value for the total fixed loss can be obtained directly from the results of three simple tests.

EXAMPLE 5.3. The results of a locked rotor and running light tests on a 415 V, 30 kW, 3-phase, 50 Hz, delta-connected squirrel-cage induction motor are:

	Line voltage	*Line current*	*Total power*
Locked Rotor	130 V	77 A	6·4 kW
Running Light	415 V	22·8 A	1·4 kW

The stator resistance is 0·48 ohm per phase. Determine the parameters per phase of the equivalent circuit of the motor.

Solution: The stator resistance R_1 is given as 0·48 ohm per phase. From the running light test, the total fixed losses P_f are given by (5.26) as

$$P_f = 1400 - 3 \times 0{\cdot}48 \times \left(\frac{22{\cdot}8}{\sqrt{3}}\right)^2 = 1150\ \text{W}.$$

The input impedance on no-load is given by

$$Z_{oc} = \frac{415}{22{\cdot}8/\sqrt{3}} = 31{\cdot}4\ \text{ohms}.$$

The no-load resistance R_{oc} will be given by

$$R_{oc} = \frac{1400/3}{(22{\cdot}8/\sqrt{3})^2} = 2{\cdot}7 \text{ ohms.}$$

Then, from (5.27)

$$X_{oc} = X_1 + X_m = \sqrt{(31{\cdot}4^2 - 2{\cdot}7^2)^{\frac{1}{2}}}, \quad \text{i.e., } X_1 + X_m = 31{\cdot}3 \text{ ohms.}$$

From the locked rotor test $P_{sc} = mI_{sc}^2(R_1 + R_{2_1})$. That is,

$$\frac{6400}{3} = \left(\frac{77}{\sqrt{3}}\right)^2 (0{\cdot}48 + R_{2_1}), \quad \text{whence} \quad R_{2_1} = 0{\cdot}60 \text{ ohm.}$$

The input impedance on short-circuit is given by

$$Z_{sc} = \frac{130}{77/\sqrt{3}} = 2{\cdot}92 \text{ ohms.}$$

Then from (5.29),

$$X_1 + X_{2_1} = [2{\cdot}92^2 - 1{\cdot}06^2]^{\frac{1}{2}} = 2{\cdot}72 \text{ ohms.}$$

For a standard squirrel-cage motor it can be assumed that $X_1 = X_{2_1}$ and $X_1 = X_{2_1} = 1{\cdot}36$ ohm. Thus $X_m = 31{\cdot}3 - X_1 = 29{\cdot}94$.

5.6. Starting methods for induction motors

Direct-on starting

The impedance of a polyphase induction motor at standstill is relatively small and when such a machine is connected directly to a supply system, the initial current will be high (of the order of *six* times full-load current) and at a low power-factor. The size of machine which can be direct-on starting will depend on the capacity of the available supply system and this method of starting is only suitable for relatively small (up to 7·5 kW) machines. The performance of the machine at standstill can be determined from one of the usual equivalent circuits.

Delta star starting

The machine is designed for delta operation and is connected in star during the starting period. The impedance between line terminals for star connection will be three times that for delta connection and for the same line voltage the line current at standstill for star connection will be reduced to one-third of the value for delta connection. Since the phase voltage will be reduced by a factor $\sqrt{3}$ during starting, it follows that the starting torque will be one-third of normal.

However, when the machine connection is changed from star to delta, all three switches must be opened simultaneously. During this period the air-gap flux and the speed will decrease and, when the switches are reclosed, the transient current surge can be high. Thus, this method of starting does not necessarily reduce the peak value of starting current but should reduce the time duration of this current.

Auto-transformer starting

The connection for auto-transformer starting is shown in Fig. 5.12 and the setting of the auto-transformer can be predetermined to limit the starting current to any desired value. An auto-transformer which reduces the voltage

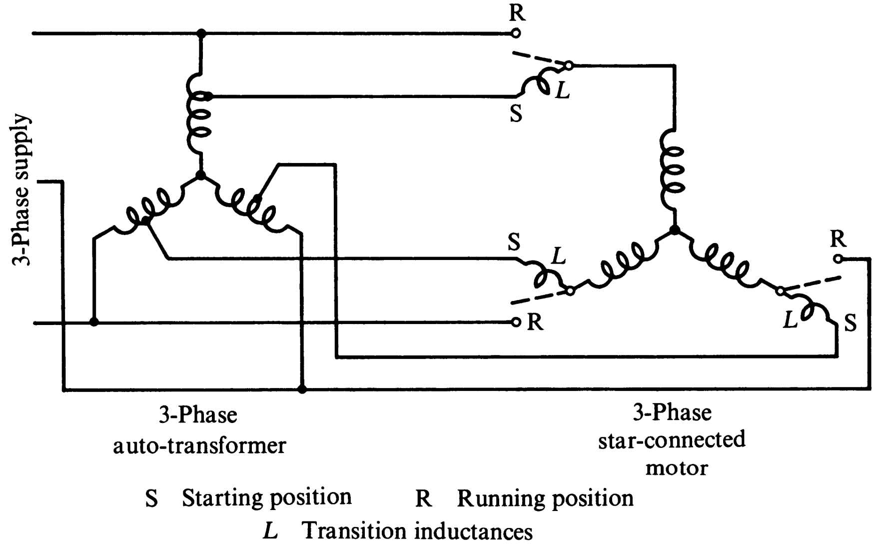


Fig. 5.12. Auto-transformer starting.

applied to the motor to x times normal voltage will reduce the starting current in the supply system and the starting torque of the motor to x^2 times normal values. This method can be applied to both star- and delta-connected machines but suffers from the same disadvantage as star-delta starting in that all three line switches must be opened simultaneously during the change-over from the starting to the normal running condition. However, the current surge during switching can be reduced by introducing transition impedances as shown in Fig. 5.12.

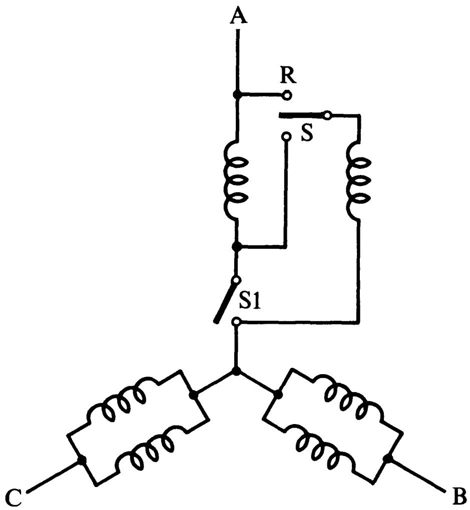
Rotor resistance starting

It is obvious that the starting current can be reduced if the motor impedance is increased and, in the particular case of the wound rotor machine, this can be done by introducing additional rotor resistance during the starting period. This can also be made to increase the starting torque available since the slip at which maximum torque occurs increases as the rotor resistance increases.

Thus, high values of starting torque per ampere of starting current can be obtained using this method. The rotor resistance will generally be reduced in steps as the machine runs up to speed.

Multi-circuit starting (ref. 3)

The circuit connections corresponding to this method of starting are shown in Fig. 5.13 and it can be seen that the motor is connected asymmetrically



Switch S1 open for starting (S)
closed for running (R)

Fig. 5.13. Multi-circuit starting.

during the starting period with usually twice as many turns in series in one phase as in the other two phases. This will, of course, result in unbalanced currents in the supply system but the advantage of this method is that the starting torque per ampere of starting current is high and the mean starting

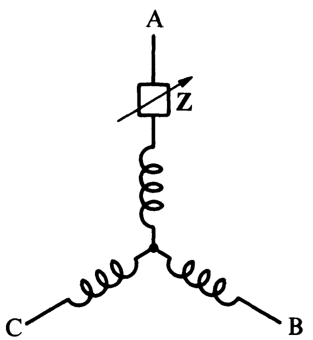


Fig. 5.14. Kusa method of starting.

current is low. The main advantage of this method, however, is that during the transition from the starting connection to the running connection, only one phase of the machine is open-circuited and the machine will continue to produce torque as a single-phase motor during the transition period.

A starting method, similar to this starting method, which is known as the Kusa method of starting, is illustrated in Fig. 5.14, and consists of inserting a variable impedance in one supply line to the machine during the run-up period.

5.7. Speed control of induction motors

The simple squirrel-cage induction motor is ideally suited to drives where an approximately constant speed is required. Where several discreet speeds or a continuously variable speed is required, modifications can be made to the induction motor to suit it to such applications. Variable speed can be produced in one of two ways. First, the synchronous speed of the machine can be changed by altering the frequency of the supply or the number of poles. Second, the shape of the torque–speed curve can be changed by varying the applied voltage or by injecting power into the secondary of the machine. The various methods by which the speed of the polyphase induction motor can be controlled will be discussed in this section.

(a) Pole amplitude modulation

A general method of pole changing, known as pole amplitude modulation (ref. 4), has been recently devised, which enables a single-winding squirrel-cage induction motor to run equally well at either of two chosen speeds. This new method is based on the fact that, when an m.m.f. sinusoidally distributed in space, with a given number of poles, is multiplied by another space-distributed sinusoid of a different period, the resulting m.m.f. can be considered as two separate m.m.f.s with different pole numbers. For a particular instant in time, the m.m.f. F, produced by the current in the phase winding of a 3-phase machine can be written as

$$F = A \sin \frac{p\theta}{2} \tag{5.30}$$

where A is the amplitude of the m.m.f., p is the number of poles and θ°m is the mechanical angle in radians around the stator periphery.

When the amplitude A is modulated in space, in such a way that $A = C \sin K\theta$, where K is an integer, (5.30) can be written

$$F = C \sin K\theta \sin \frac{p\theta}{2} = \frac{C}{2}\left[\cos\left(\frac{p}{2} - K\right)\theta + \cos\left(\frac{p}{2} + K\right)\theta\right]. \tag{5.31}$$

Thus, from (5.31), the resulting phase m.m.f. can be said to consist of two separate m.m.f.s with different pole numbers, $(p + 2K)$ and $(p - 2K)$. It is, of course, necessary to eliminate one of the pole numbers produced by modulation, and the original method consisted of modulating each phase separately, the individual phase windings being arranged in such a manner that one of the pole numbers produced by modulation is eliminated from the total m.m.f. of the winding.

As a particular case, consider the 8-pole arrangement ($p = 8$) shown in Fig. 5.15(a) modulated by the rectangular 2-pole wave ($K = 1$) shown in Fig. 5.15(b). The process of modulation can be simply performed by reversing

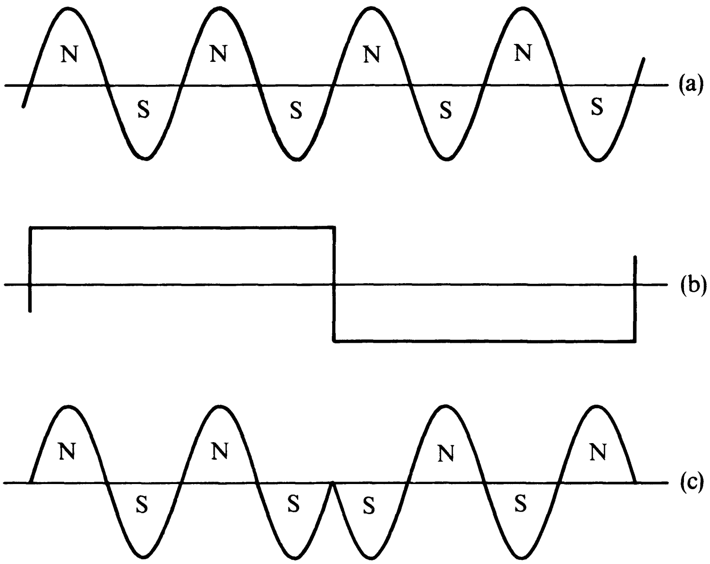


Fig. 5.15. Basis of pole amplitude modulation:
(a) Basic 8-pole m.m.f.
(b) Modulating wave.
(c) Modulated m.m.f.

one-half of the phase winding relative to the other half and the resulting m.m.f. shown in Fig. 5.15(c) will consist of a 6-pole $(p - 2K)$ and a 10-pole $(p + 2K)$ m.m.f. If the 3-phase, 8-pole winding is arranged with the phase entries 120°m apart in space, i.e., as if it were a 2-pole winding, the 6-pole m.m.f. produced by modulation will represent the third harmonic of the original winding. It has been shown in chapter 3 that the total m.m.f. produced by a symmetrical 3-phase winding will contain no triple harmonics, so that the total 6-pole m.m.f. produced by modulation will vanish and the machine will operate with a 10-pole m.m.f., which can be considered as the fifth harmonic of a basic 2-pole wave.

Since, in the 10-pole mode, the machine operates on what is essentially an exaggerated fifth harmonic, the presence of appreciable space-harmonics in the total m.m.f. could be of considerable importance, and the basic modulation method described in Fig. 5.15 is likely to produce undesirable crawling effects which can, however, be reduced by chording the winding. A very great improvement in performance can be brought about by the use of windings initially of the fractional slot type. In the case of a 36-slot 8/10 pole machine, the unconventional form of fractional slot winding used is given in Table 5.2. The coil pitch is 2/3 (pole pitch) for the 8-pole connection and 5/6

TABLE 5.2

Table of number of coils per phase belt for 8/10 pole-changing

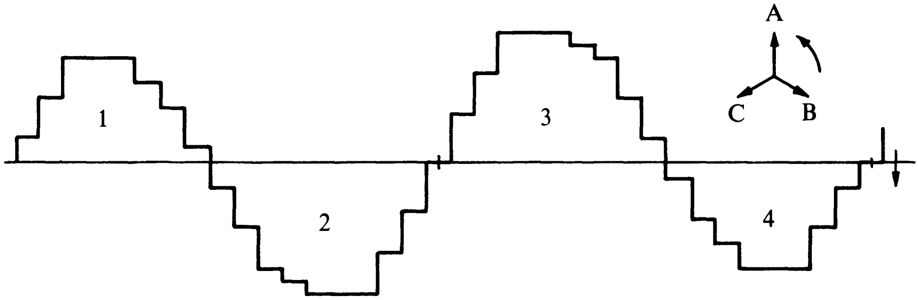
POLES

	1	2	3	4	5	6	7	8
Phase A	2	4	4	2	2	4	4	2
Phase B	2	2	4	4	2	2	4	4
Phase C	4	2	2	4	4	2	2	4
Total	8	8	10	10	8	8	10	10

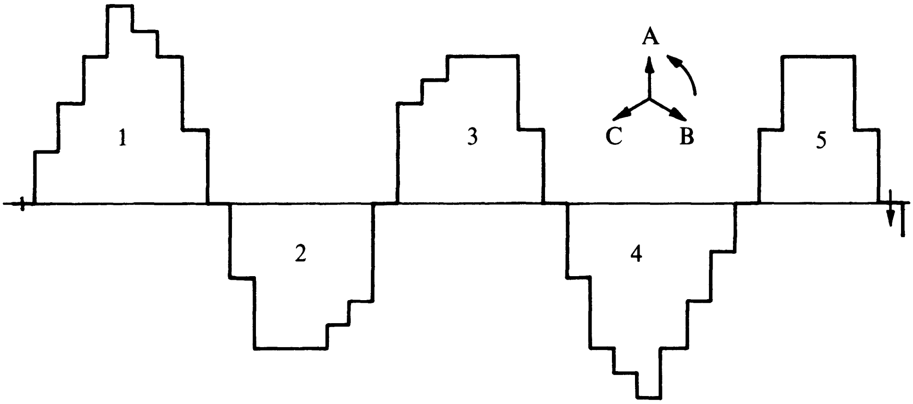
(pole pitch) for the 10-pole connection. Modulation is effected by reversal of one-half of each of the phase windings, and the space distribution of the total m.m.f. for a particular instant in time, before and after modulation, is shown in Fig. 5.16. The use of this technique will produce a great improvement in the performance of the machine with the modulated pole number, with little reduction in performance for the unmodulated case.

The basic method, for which each phase-winding is considered separately, precludes pole numbers which are a multiple of three; so that, for example, while an 8/10 pole arrangement was possible, an 8/6 pole one was not. Later developments in the technique (refs. 5, 6) have led to the extension of the available range to include, first, any pole-ratio between 1 and 1·5 in the close ratio range, and finally all other wide-ratio pole numbers. In all cases only six external control leads are required and the method of pole-amplitude modulation has, in effect, turned the standard single-winding squirrel-cage induction motor into a two-speed machine with performance at each speed closely approaching single-speed standards, using methods which are simple and inexpensive.

The squirrel-cage induction motor long ago became the predominant industrial electrical motor, and it has achieved this because of its many outstanding merits (cheapness, reliability, good performance, absence of sliding contacts, etc.) and in spite of its one great defect—a fixed speed. In many cases, a fixed speed is quite sufficient, in some cases, it is tolerated for simplicity; in other cases, a change in speed is obtained by gears or other mechanical devices. The method of pole-amplitude modulation has added a new degree of freedom to the induction motor and the motors so far in operation have been applied to pumps, fans, grinders, crushers, escalators, conveyor belts, and machine tools.



(a) Before modulation



(b) After modulation

Fig. 5.16. M.m.f. waveforms for fractional-slot arrangement: (a) Before modulation. (b) After modulation.

Most textbooks consider a 2/1 pole-changing winding with six terminals, invented over 60 years ago and manufactured all over the world. This winding is, in fact, a particular example of pole-amplitude modulation devised without recognition of the general principle behind its action. Since this one speed-ratio alone has found many and wide uses, it seems almost certain that a general method of pole-changing which offers any speed-ratio will be used even more extensively.

(*b*) *Variation of supply voltage and frequency*

The torque developed by the machine at a particular slip is proportional to the square of the applied voltage and the shape of the torque–speed curve can be modified by control of the applied voltage.

When the frequency of the voltage applied to the machine is varied, the synchronous speed is changed and, if the applied voltage is controlled directly with the frequency to maintain approximately constant air-gap flux density, torque–speed characteristics of the form shown in Fig. 5.17 are

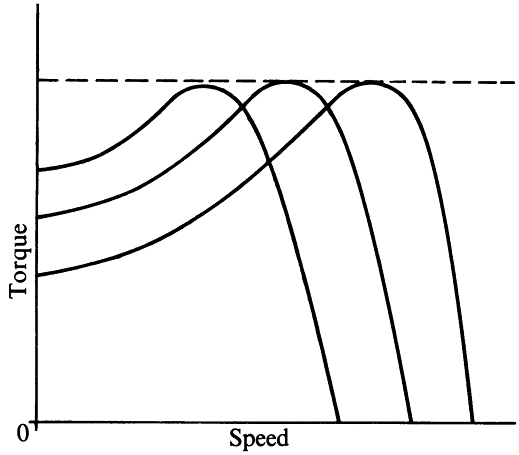


Fig. 5.17. Torque–slip curves with variable frequency.

obtained. The major disadvantage of this method of speed control is that it involves the use of a polyphase variable frequency power supply. However, the discovery of the thyristor (a controlled silicon rectifier), which is capable of providing large powers at variable frequency from a fixed frequency source, could do much to overcome this difficulty.

(*c*) *Variation of rotor resistance*

In the particular case of a wound-rotor machine, the effective resistance of the secondary circuit can be controlled by the use of external variable resistances. The form of the torque–speed curve with varying rotor resistance, derived in section 5.3, is shown in Fig. 5.9, and it can be seen that, for a given load, the operating speed of the machine varies with the value of the rotor resistance. However, the rotor copper loss is always high at large slips, so that this method of speed control can be very inefficient.

(*d*) *Rotor injection using commutator machines*

The action of the commutator as a frequency changer has been discussed in chapter 1, and the use of a polyphase commutator machine connected to the slip-rings of a wound-rotor induction motor, injecting power into the secondary, to produce speed control has been mentioned. Various forms of commutator machines and their application to the speed and power-factor control of polyphase a.c. motors will be described in the following sections.

The simplest form of polyphase commutator machine is illustrated in Fig. 5.18. It consists of a lap-connected rotor winding connected to a commutator with an unwound stator. When 3-phase voltages of frequency $f = \omega_s/2\pi$

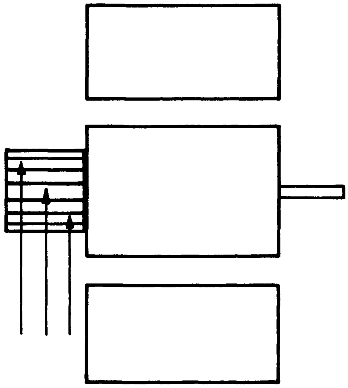


Fig. 5.18. Basic form of polyphase commutator machine.

are applied to the commutator, its action is to frequency change these voltages to a frequency $f_2 = (\omega_s \pm \omega)/2\pi$, where ω is the mechanical speed of rotation. Since the rotor winding is inductive, the equivalent impedance of the commutator will change in phase and magnitude as the speed ω is changed. When the commutator brushes are connected to the slip-rings of the wound-rotor induction motor under control, the commutator machine effectively injects voltage into the secondary of the main motor. It is important to note that this injected voltage will be proportional to the secondary

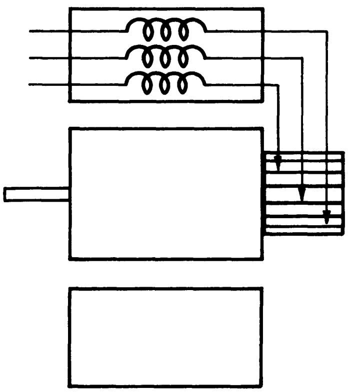


Fig. 5.19. Polyphase commutator machine with stator winding.

current of the motor under control and this method is only effective when the main motor is operating on load with a relatively high secondary current.

A considerable improvement to the performance of this arrangement can be made by the use of a stator winding connected in series with the commutator as shown in Fig. 5.19. This arrangement will also produce an injected voltage proportional to the secondary current of the motor under control and has the advantage that the phase angle of the voltage can be controlled by the position of the commutator brushes relative to the stator winding. A second stator winding is now necessary to compensate for the effects of armature reaction.

When the main stator winding is connected in parallel with the commutator brushes, an injected voltage independent of the value of the secondary current of the machine under control is obtained and this arrangement, known as the Scherbius machine, is shown in Fig. 5.20.

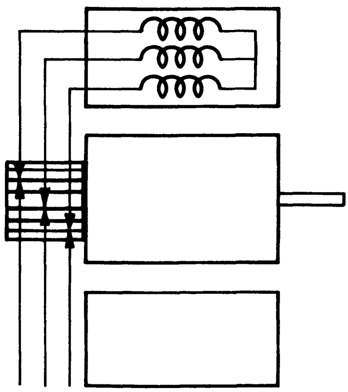


Fig. 5.20. The Scherbius machine.

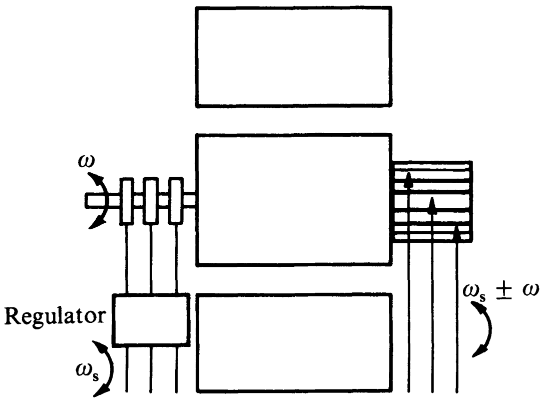


Fig. 5.21. The frequency changer.

The most elegant form of polyphase commutator machine is the Frequency Changer, described in chapter 1 and illustrated in Fig. 5.21. The injected voltage will be independent of the secondary current of the machine under control, its magnitude will be governed by the setting of the regulator and its phase angle will be controlled by the brush position on the commutator.

5.8. Dynamic operation of induction motors

An induction motor operates under dynamic conditions when the speed of the motor changes from the steady-state value to another. A change in speed will result from a change in the voltage applied to the motor or from a change in load conditions when the machine is operating under stable conditions. Any electrical transients associated with a change in speed will be neglected in the following analysis.

The power balance for a polyphase induction motor has been considered in section 5.2, and, from this analysis it follows that

$$\text{Torque in synchronous watts} = mI_{21}^2 R_{21}/S = \omega_s T_e$$

where ω_s is the synchronous speed and T_e N.m is the torque of the m-phase machine.

The rotor copper loss P_r is given by

$$P_r = S(\omega_s T_e) \tag{5.32}$$

where $S = (\omega_s - \omega)/\omega_s$ is the slip when the rotor speed is ω.

With the machine on no-load

$$T_e = J\frac{d\omega}{dt} = J\frac{d}{dt}\,\omega_s(1 - S) = -\omega_s J\frac{dS}{dt} \tag{5.33}$$

so that from (5.32) and (5.33),

$$P_r = -J\omega_s^{\,2}\,\frac{S\,dS}{dt}.$$

Then the energy dissipated in the rotor when the slip changes from S_1 to S_2 is given by

$$\text{Energy} = \int P_r\,dt = \int_{S_1}^{S_2} -J\omega_s^{\,2} S\,dS.$$

Thus,
$$\text{Energy} = \frac{J\omega_s^{\,2}}{2}(S_1^{\,2} - S_2^{\,2})\text{ joules.} \tag{5.34}$$

When an ideal polyphase induction motor accelerates from standstill to the synchronous speed ω_s, the change in kinetic energy of the rotating mass is given by: kinetic energy $= \frac{1}{2}J\omega_s^{\,2}$.

The corresponding energy dissipated in the rotor winding is given from (5.34) as: rotor energy loss $= \frac{1}{2}J\omega_s^2$.

For an ideal machine, the input energy is given by

$$\text{Input energy} = \text{Kinetic energy} + \text{Rotor energy} = J\omega_s^2.$$

Under these conditions the energy efficiency is given by

$$\text{Energy efficiency} = \frac{\text{Output energy}}{\text{Input energy}} = 50\%.$$

Thus the maximum possible efficiency on an induction motor when the speed is changing is 50%.

When the stator resistance can be neglected, the relationship between the machine torque and the slip is given in (5.22) as

$$T_e = T_m \frac{2}{\dfrac{S}{S_t} + \dfrac{S_t}{S}} \tag{5.35}$$

where T_m is the maximum torque and S_t is the slip for maximum torque. Then from (5.33) and (5.35)

$$\frac{2T_m}{\dfrac{S}{S_t} + \dfrac{S_t}{S}} = -\omega_s J \frac{dS}{dt}, \quad \text{or} \quad \frac{2T_m}{\omega_s J} dt = -\left(\frac{S}{S_t} + \frac{S_t}{S}\right) dS.$$

The time t taken for the slip to change from S_1 to S_2 is given by

$$\frac{2T_m}{\omega_s J} t = -\int_{S_1}^{S_2} \left(\frac{S}{S_t} + \frac{S_t}{S}\right) dS$$

or

$$t = \frac{\omega_s J}{2T_m} \left[\frac{S_1^2 - S_2^2}{2S_t} + S_t \log_e \frac{S_1}{S_2}\right]. \tag{5.36}$$

When the load torque is constant or is a linear function of speed, the methods of analysis described in this section can be applied directly.

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Tutorial Problems

1. An 8-pole, 200 V, 50 Hz, 3-phase induction motor runs at 726 r.p.m. Calculate the slip r.p.m. per-unit slip and the frequency of the rotor current. If the machine develops 5·5 h.p. gross at this speed, calculate the rotor copper loss and torque in N.m.

(*Answer:* 24 r.p.m.; 0·032; 1·6 c/s; 136 W; 54 N.m)

2. A 6-pole, 400 V, 50 Hz, 3-phase induction motor has an output of 30 b.h.p. at a slip of 0·02 p.u. If the torque required for mechanical losses is 20 N.m and the stator loss is 0·9 kW, calculate the input power and efficiency.

(*Answer:* 25·9 kW; 0·865)

3. A 415 V, 50 Hz, 3-phase, 4-pole induction motor develops its full load power of 10 h.p. at 1425 r.p.m. and the input is 10 kW at power-factor 0·8 lagging. On no-load the input is 0·5 kW at rated voltage. Calculate the full-load slip, line current, stator copper loss, rotor copper loss and torque in N.m.

(*Answer:* 0·05; 17·2 A; 1·62 kW; 0·419 kW; 50·1 N.m)

4. The parameters per phase referred to the primary of a 200 V, 3-phase, 4-pole, 50 Hz star-connected induction motor are as follows:

$$R_1 = 0{\cdot}11\ \Omega;\quad X_1 = 0{\cdot}35\ \Omega;\quad R_{21} = 0{\cdot}13\ \Omega;\quad X_{21} = 0{\cdot}35\ \Omega;\quad X_m = 14\ \Omega.$$

Calculate the percentage error involved when the maximum torque of the machine is determined, neglecting stator impedance.

(*Answer:* 142%)

5. A 415 V, 50 Hz, 25 h.p., 6-pole, 3-phase, star-connected induction motor is started with an external resistance of 2 ohm in each supply line. The parameters of its equivalent circuit are:

$$R_1 = 0{\cdot}25\ \Omega;\quad X_1 = 0{\cdot}75\ \Omega;\quad R_2 = 0{\cdot}17\ \Omega;\quad X_2 = 0{\cdot}49\ \Omega;\quad X_m = 32\ \Omega.$$

If the stator to rotor phase voltage ratio is 415/365, calculate the starting torque developed and the corresponding operating power-factor for direct-on starting.

(*Answer:* 44·3 N.m; 0·872)

6. A squirrel-cage induction motor with negligible stator resistance produces a starting torque of 150% of the full-load torque and a maximum torque of 250% of the full-load torque. Calculate the slip for maximum torque and the slip for full-load torque.

(*Answer:* 0·33; 0·071)

7. The reactance of the stator of a 3-phase induction motor is equal to the referred value of the rotor reactance at standstill while each resistance is one-fifth of the reactance. Find the values of the starting torque and the maximum torque if the full-load torque of 520 N.m occurs at a slip of 0·04. Neglect no-load current.

(*Answer:* 200 N.m; 928 N.m)

8. A 200 V, 50 Hz, 6-pole, 3-phase, star-connected squirrel-cage induction motor gave the following test results:

	Line voltage	*Line current*	*Total power*
Open-circuit	200 V	7·9 A	520 W
Short-circuit	133 V	48 A	5540 W

The d.c. resistance per phase of the stator is 0·42 Ω and the stator and the referred rotor reactances are equal. Derive the parameters of the equivalent circuit.
(*Answer:* $R_1 = 0{\cdot}42\ \Omega$; $X_1 = X_{21} = 0{\cdot}692\ \Omega$; $R_{21} = 0{\cdot}382\ \Omega$; $X_m = 13{\cdot}5\ \Omega$)

9. A 6-pole, 400 V, 50 Hz, 3-phase induction motor has a star-connected stator and rotor. The stator impedance is $0{\cdot}6 + j1{\cdot}5\ \Omega$ per phase, the equivalent rotor impedance at standstill is $0{\cdot}6 + j2{\cdot}0\ \Omega$ per phase and the impedance of the magnetizing branch is $j35\ \Omega$. During starting, the rotor is connected to a star-connected equivalent impedance of $1 + j0{\cdot}2\ \Omega$ per phase. Determine the starting current and torque. If full load occurs at a slip of 0·05, find the normal full-load current and the b.h.p. allowing a mechanical loss of 300 W.
(*Answer:* 55·2 A; 171 N.m; 19 A; 4·05 h.p.)

10. A 440 V, 50 Hz, 15 h.p., 6-pole, star-connected wound rotor induction motor has the following parameters per phase:

$$R_1 = 0{\cdot}9\ \Omega;\quad R_{21} = 0{\cdot}8\ \Omega;\quad X_1 = X_{21} = 2{\cdot}0\ \Omega;\ X_m = 48\ \Omega.$$

The no-load losses are 494 W. Calculate
(a) the starting torque for direct-on starting.
(b) the maximum torque and corresponding slip.
(c) the maximum output power and corresponding slip.
(d) the values of external referred rotor resistance required to produce two-thirds of maximum torque at standstill.
(*Answer:* 74·5 N.m; 187 N.m; 0·198; 14·56 kW, 0·158; 0·59 Ω or 10·66 Ω)

11. A 3-phase, 4-pole, 50 Hz, wound-rotor induction motor develops its maximum torque of 250% of full-load torque at a slip of 0·2 when operating at rated voltage. Determine the minimum value of per-unit applied voltage at which the machine will supply full-load torque, and the external referred value of rotor resistance required to produce maximum starting torque at standstill with the reduced applied voltage, expressed as a percentage of the referred rotor resistance/phase of the machine.
(*Answer:* 63·3%; 400%)

12. Find the ratio of starting to full-load current for a 400 V, 15 h.p., 3-phase induction motor fitted with a star-delta starter if the full-load power factor is 0·85, the full-load efficiency is 88% and the short-circuit current is 40 A at 200 V.
(*Answer:* 1·25)

13. An 8-pole, 50 Hz, 3-phase, induction motor has a rotor resistance of 0·04 ohm per phase referred to the primary and the maximum torque occurs at 645 r.p.m. Determine the ratio of starting torque to maximum torque if (a) direct-on starting is used, (b) an auto-transformer with a 70% tapping is used for starting. Stator resistance can be neglected.
(*Answer:* 0·275; 0·1335)

14. (a) Show that the maximum possible efficiency of polyphase induction motor during run-up from standstill is 50%.
(b) A 3-phase, 50 Hz, 4-pole induction motor runs on no-load with a slip of 1% and produces a torque of 40 N.m at a slip of 10%. The machine drives a load of total moment of inertia 5 kg.m^2 and negligible friction. Determine an expression for the speed of the machine following the application of a step load of 20 N.m, if the torque–speed curve of the motor can be assumed to be linear over its working range.
(*Answer:* $1425 + 60\ e^{-0{\cdot}51t}$)

6. Single-phase motors

Many small a.c. motors will be designed to operate from a single-phase supply and will take the form of single-phase induction motors or a.c. commutator motors. The single-phase squirrel-cage induction motor is the most widely used form of machine but suffers from the disadvantage, noted in chapter 1, that it produces no starting torque. In order to produce starting torque, a second stator winding will normally be introduced so that, during the starting period, the machine operates as an asymmetrical 2-phase motor connected to a single-phase supply. In these circumstances, the operation of the symmetrical polyphase machine on a single-phase system can be used to illustrate the principles underlying the performance of single-phase induction motors.

6.1. The single-phase induction motor

It has previously been noted that most single-phase induction motors are, in fact, 2-phase motors, in which one winding, the so-called auxiliary winding, is disconnected from the supply when the speed reaches a certain value and the machine then operates as a true single-phase motor with a single stator winding, the so-called main winding. The connection diagram for such a machine is shown in Fig. 6.1, in which the phase converter **Y** can be resistive,

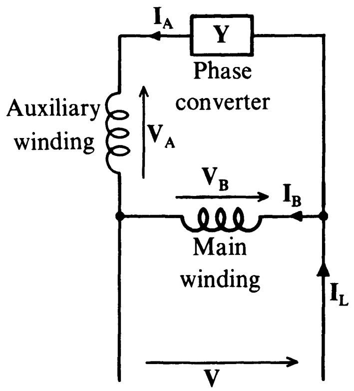


Fig. 6.1. Single-phase induction motor.

capacitive, or inductive. This machine can be analysed (ref. 1) by the method of 2-phase symmetrical components and the unbalanced 2-phase system vectors $\mathbf{U}_A$ and $\mathbf{U}_B$ can be written

$$\mathbf{U}_B = \mathbf{S}_p + \mathbf{S}_n \tag{6.1}$$

$$\mathbf{U}_A = j\mathbf{S}_p - j\mathbf{S}_n \tag{6.2}$$

so that

$$\mathbf{S}_p = \tfrac{1}{2}(\mathbf{U}_B - j\mathbf{U}_A) \tag{6.3}$$

$$\mathbf{S}_n = \tfrac{1}{2}(\mathbf{U}_B + j\mathbf{U}_A). \tag{6.4}$$

These equations can only be applied to a symmetrical machine and, in practice, the number of turns on the auxiliary phase winding will be different to that on the main phase winding. Ideally, two windings having turns in the ratio $K:1$ should have impedances in the ratio $K^2:1$. Thus, if the admittance of the main winding is $\mathbf{Y}_m$, the ideal admittance of the auxiliary phase is $\mathbf{Y}_m/K^2$ and any difference between the actual admittance $\mathbf{Y}_a$ and the ideal admittance $\mathbf{Y}_m/K^2$ of the auxiliary winding is assumed to be part of the external phase balance $\mathbf{Y}$. Under these conditions the symmetrical component equations must be written with reference to the connection given in Fig. 6.1 as

$$\mathbf{V}_B = \mathbf{V}_p + \mathbf{V}_n, \qquad \mathbf{I}_B = \mathbf{I}_p + \mathbf{I}_n \tag{6.5}$$

$$\mathbf{V}_A = K(j\mathbf{V}_p - j\mathbf{V}_n), \qquad \mathbf{I}_A = \frac{1}{K}(j\mathbf{I}_p - j\mathbf{I}_n) \tag{6.6}$$

such that $\mathbf{I}_p = \mathbf{V}_p\mathbf{Y}_p$ and $\mathbf{I}_n = \mathbf{V}_n\mathbf{Y}_n$.

Two inspection equations can be obtained from Fig. 6.1 in the form

$$\mathbf{V} = \mathbf{V}_B \tag{6.7}$$

$$\mathbf{V} = \mathbf{V}_A + \frac{\mathbf{I}_A}{\mathbf{Y}}. \tag{6.8}$$

When the values for the sequence quantities from (6.5) and (6.6) are substituted in (6.7) and (6.8) it can be shown that

$$\frac{\mathbf{V}_p}{\mathbf{V}} = \frac{\mathbf{Y}_n + K(K - j)\mathbf{Y}}{2K^2\mathbf{Y} + \mathbf{Y}_p + \mathbf{Y}_n} \tag{6.9}$$

$$\frac{\mathbf{V}_n}{\mathbf{V}} = \frac{\mathbf{Y}_p + K(K + j)\mathbf{Y}}{2K^2\mathbf{Y} + \mathbf{Y}_p + \mathbf{Y}_n}. \tag{6.10}$$

In the particular case of standstill, $\mathbf{Y}_p = \mathbf{Y}_n = \mathbf{Y}_s$, so that

$$\frac{\mathbf{V}_p}{\mathbf{V}} = \frac{\mathbf{Y}_s + K(K - j)\mathbf{Y}}{2(K^2\mathbf{Y} + \mathbf{Y}_s)} \tag{6.11}$$

$$\frac{\mathbf{V}_n}{\mathbf{V}} = \frac{\mathbf{Y}_s + K(K + j)\mathbf{Y}}{2(K^2\mathbf{Y} + \mathbf{Y}_s)} \tag{6.12}$$

and

$$T_e = A_s(V_p^2 - V_n^2). \tag{6.13}$$

It should be noted that, since at standstill there will be no coupling between the two stator phase windings, the equivalent circuit of Fig. 6.1 can be redrawn in the manner shown in Fig. 6.2. The phase and line quantities can be obtained directly from Fig. 6.2 so that

$$\mathbf{V}_A = \mathbf{V}\frac{K^2\mathbf{Y}}{\mathbf{Y}_s + K^2\mathbf{Y}} \tag{6.14}$$

$$\mathbf{V}_{pc} = \frac{\mathbf{V}\mathbf{Y}_s}{\mathbf{Y}_s + K^2\mathbf{Y}} \tag{6.15}$$

$$\mathbf{V}_B = \mathbf{V} \tag{6.16}$$

$$\mathbf{I}_A = \mathbf{V}\frac{\mathbf{Y}\mathbf{Y}_s}{\mathbf{Y}_s + K^2\mathbf{Y}} \tag{6.17}$$

$$\mathbf{I}_B = \mathbf{V}\mathbf{Y}_s \tag{6.18}$$

$$\mathbf{I}_L = \frac{\mathbf{V}\mathbf{Y}_s}{\mathbf{Y}_s + K^2\mathbf{Y}}[\mathbf{Y}_s + \mathbf{Y}(1 + K^2)]. \tag{6.19}$$

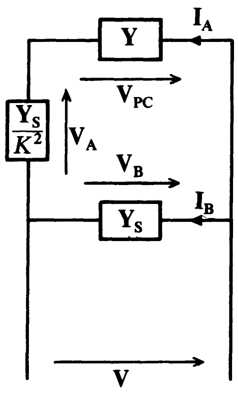


Fig. 6.2. Circuit for single-phase induction motor.

Considerable simplification in the analysis can be made by the use of a normalization technique, by putting

$$\frac{\mathbf{Y}_s}{\mathbf{Y}} = \frac{Y_s\underline{|\phi}}{Y\underline{|\beta}} = y\overline{\backslash\alpha} = y(\cos\alpha - j\sin\alpha). \tag{6.20}$$

The sequence voltages given by (6.11) and (6.12) can now be written

$$\frac{\mathbf{V}_p}{\mathbf{V}} = \frac{(y\cos\alpha + K^2) - j(y\sin\alpha + K)}{2(y\cos\alpha + K^2) - 2jy\sin\alpha} \tag{6.21}$$

$$\frac{\mathbf{V}_n}{\mathbf{V}} = \frac{(y\cos\alpha + K^2) - j(y\sin\alpha - K)}{2(y\cos\alpha + K^2) - 2jy\sin\alpha}. \tag{6.22}$$

The torque ratio t is given by

$$t = \frac{A_s(V_p^2 - V_n^2)}{A_s V^2} = \frac{Ky \sin\alpha}{y^2 + 2K^2 y \cos\alpha + K^4}. \qquad (6.23)$$

Equation (6.23) expresses the starting torque ratio t in terms of three dimensionless parameters y, α, and K, so that the variations of the torque ratio with variation in each of these parameters separately can be fully investigated. In many cases the form of the external phase converter and the turns ratio will be specified and only the effects of variation of the parameter y will be investigated further. From (6.23) the condition for maximum starting torque is given by

$$\frac{dt}{dy} = K \sin\alpha \left[\frac{y^2 + 2K^2 y \cos\alpha + K^4 - y(2y + 2K^2 \cos\alpha)}{(y^2 + 2K^2 y \cos\alpha + K^4)^2}\right] = 0$$

so that

$$y = K^2 \qquad (6.24)$$

is the condition for maximum starting torque t_m and

$$t_m = \frac{\sin\alpha}{2K(1 + \cos\alpha)}. \qquad (6.25)$$

In the particular case of a machine with a main winding standstill impedance angle of 45°, $\phi = -45°$. Then, for a capacitive phase converter, $\beta = 90°$ and (6.25) can be written

$$t_{mc} = \frac{\sin 135°}{2K(1 + \cos 135°)} = \frac{1{\cdot}21}{K}. \qquad (6.26)$$

Similarly, for a resistance phase converter $\beta = 0°$, so that

$$t_{mr} = \frac{\sin 45°}{2K(1 + \cos 45°)} = \frac{0{\cdot}209}{K}. \qquad (6.27)$$

It immediately follows that, for a given K, a phase converter in the form of a capacitor will produce considerably more starting torque than one in the form of a resistor.

The imbalance factor U is the ratio of the magnitude of the negative sequence current to that of the positive sequence current so that, at standstill,

$$U = \frac{I_n}{I_p} = \frac{V_n}{V_p} = \left[\frac{y^2 + 2Ky(K\cos\alpha - \sin\alpha) + K^2(K^2 + 1)}{y^2 + 2Ky(K\cos\alpha + \sin\alpha) + K^2(K^2 + 1)}\right]^{\frac{1}{2}} \qquad (6.28)$$

and it can be shown that minimum imbalance will occur when

$$y = K\sqrt{(1 + K^2)}. \qquad (6.29)$$

This value of y will also produce maximum starting torque per ampere of starting current and can be taken as a suitable design criterion for the phase converter.

The phase and line quantities can also be written in normalized form so that from (6.14) and (6.15)

$$\frac{V_A}{V} = \frac{K^2}{(y^2 + 2K^2y\cos\alpha + K^4)^{\frac{1}{2}}} \tag{6.30}$$

$$\frac{V_{pc}}{V} = \frac{y}{(y^2 + 2K^2y\cos\alpha + K^4)^{\frac{1}{2}}}. \tag{6.31}$$

In a similar manner, from (6.17) and (6.19)

$$\frac{I_A}{VY_s} = \frac{1}{(y^2 + 2K^2y\cos\alpha + K^4)^{\frac{1}{2}}} \tag{6.32}$$

$$\frac{I_L}{VY_s} = \left[\frac{y^2 + 2y(1 + K^2)\cos\alpha + (1 + K^2)^2}{y^2 + 2yK^2\cos\alpha + K^4}\right]^{\frac{1}{2}}. \tag{6.33}$$

Under these conditions, the complete starting performance of a single-phase induction motor of the form illustrated in Fig. 6.1 can be obtained directly. The *split-phase motor* is the particular case for which the phase converter is in the form of a resistor and the *capacitor-start motor* is the particular case for which the phase converter is in the form of a capacitor. It follows directly from (6.26) and (6.27) that the capacitor-start motor will produce considerably more starting torque than the split-phase motor. The *capacitor-start-capacitor motor* employs a two-value capacitor to produce (i) good starting performance, using a high value of capacitance and (ii) practically balanced 2-phase operation under full-load conditions, using a low value of capacitance.

EXAMPLE 6.1. An induction motor with two stator windings in space quadrature is to be used as (a) a split-phase motor with $K = 0{\cdot}8$, (b) a capacitor-start motor with $K = 1{\cdot}3$. The standstill admittance of the main winding is $0{\cdot}1\underline{/50^\circ}$ mho. If the phase converter is set to produce minimum imbalance in each case, determine the value of the starting torque ratio, the p.u. phase converter voltage, and the line current for each machine. Determine the value of y required to limit the capacitor voltage to the supply voltage for a capacitor-start motor.

Solution: The condition for minimum imbalance is given in (6.29) as $y = K\sqrt{(1 + K^2)}$. Then

		(a) *Split phase*	(b) *Capacitor start*
	K	0·8	1·3
(6.29)	$y = K\sqrt{(1 + K^2)}$	1·02	2·13
	$\alpha = \beta - \phi$	50°	140°
(6.23)	$t = \dfrac{Ky\sin\alpha}{y^2 + 2K^2y\cos\alpha + K^4}$	0·41	0·776

		(a) Split phase	(b) Capacitor start
(6.31)	$\frac{V_{pc}}{V} = \frac{y}{(y^2 + 2K^2 y \cos\alpha + K^4)^{\frac{1}{2}}}$	0·426	1·285
(6.33)	$\frac{I_L}{VY_s} = \left[\frac{y^2 + 2y(1 + K^2)\cos\alpha + (1 - K^2)^2}{y^2 + 2yK^2\cos\alpha + K^4}\right]^{\frac{1}{2}}$	1·57	1·26
$\frac{(6.23)}{(6.33)}$	Torque per ampere	0·251	0·617

If the phase converter voltage is to be limited to be equal to the supply voltage for a capacitor-start motor, it follows from (8.55) that

$$1 = \frac{y}{(y^2 + 2K^2 y \cos\alpha + K^4)^{\frac{1}{2}}} \quad \text{or} \quad 2y\cos\alpha + K^2 = 0.$$

Then

$$y = \frac{-K^2}{2\cos\alpha} = \frac{-1{\cdot}3^2}{2\cos 140} = 1{\cdot}315.$$

6.2. Locus diagrams for single-phase motors

The starting torque produced by a single-phase motor is given by (6.13) as

$$T_e = A(V_p{}^2 - V_n{}^2) = \frac{A}{Y_s^2}(I_p{}^2 - I_n{}^2). \tag{6.34}$$

The phase currents $\mathbf{I}_M$ and $\mathbf{I}_A$ can be written as

$$\mathbf{I}_M = I_M \quad \text{and} \quad \mathbf{I}_A = I_A(\cos\lambda + j\sin\lambda).$$

Now $\mathbf{I}_M = \mathbf{I}_p + \mathbf{I}_n$ and $K\mathbf{I}_A = j\mathbf{I}_p - j\mathbf{I}_n$ so that

$$2\mathbf{I}_p = I_M + KI_A\sin\lambda - jKI_A\cos\lambda$$

$$2\mathbf{I}_n = I_M - KI_A\sin\lambda + jKI_A\cos\lambda.$$

Thus

$$I_p{}^2 - I_n{}^2 = K(I_A I_M \sin\lambda). \tag{6.35}$$

It follows directly from (6.35) that the starting torque is proportional to the product of the phase currents and the sine of the angle between them. It can also be seen from Fig. 6.1 that the line current $\mathbf{I}_L$ is the algebraic sum of the currents $\mathbf{I}_M$ and $\mathbf{I}_A$.

In the case of a split-phase motor, the reactance X_a of the auxiliary phase is constant while its resistance R_a varies. Thus the locus of the auxiliary phase current is circular and is of the form shown in Fig. 6.3. Then, for a given point P on the locus of $\mathbf{I}_A$ in Fig. 6.3, the distance AP represents $\mathbf{I}_A$ and the distance OP represents $\mathbf{I}_L$. Now PD $= I_A \sin\lambda$, so that, from (6.35), PD is proportional to torque. In a similar manner sin $\underline{|POA}$ = PD/OP and is a measure of torque per ampere of starting current.

Thus, the starting torque will be a maximum when the distance PD in Fig. 6.3 is a maximum, i.e., when the perpendicular to the line OA produced from the operating point passes through the centre C of the circle; the operating point under these conditions is P_1.

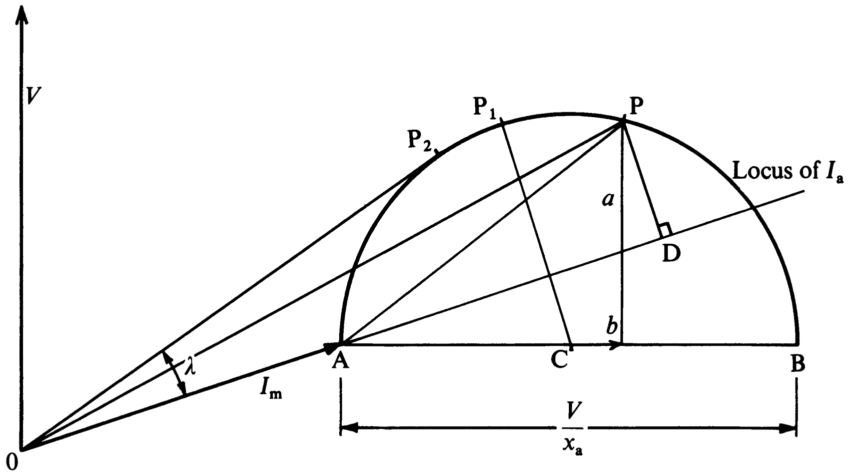


Fig. 6.3. Operating chart for split-phase motor.

In a similar manner, the starting torque per ampere will be a maximum when the angle POA is a maximum, corresponding to the operating point P_2 in Fig. 6.3 when the line OP_2 is a tangent to the circle.

For the particular operating point P in Fig. 6.3

$$\mathbf{I}_A = \frac{\mathbf{V}}{R_A + jX_A} = a - jb$$

so that, equating real and imaginary parts,

$$R_A = \frac{aX_A}{b} = \frac{V - X_A b}{a}. \tag{6.36}$$

The required value of R_A can then be obtained directly from the known value of X_A and the measured values of a and b.

In the case of a capacitor-start motor, the resistance R_A of the auxiliary phase will be constant while the total reactance varies. Thus the locus of the auxiliary phase current is again circular and is of the form shown in Fig. 6.4. The operating point for maximum starting torque is P_1 and that for maximum torque per ampere is P_2. A condition for minimum line current, corresponding to the operating point P_3 in Fig. 6.4, can also be introduced in this case.

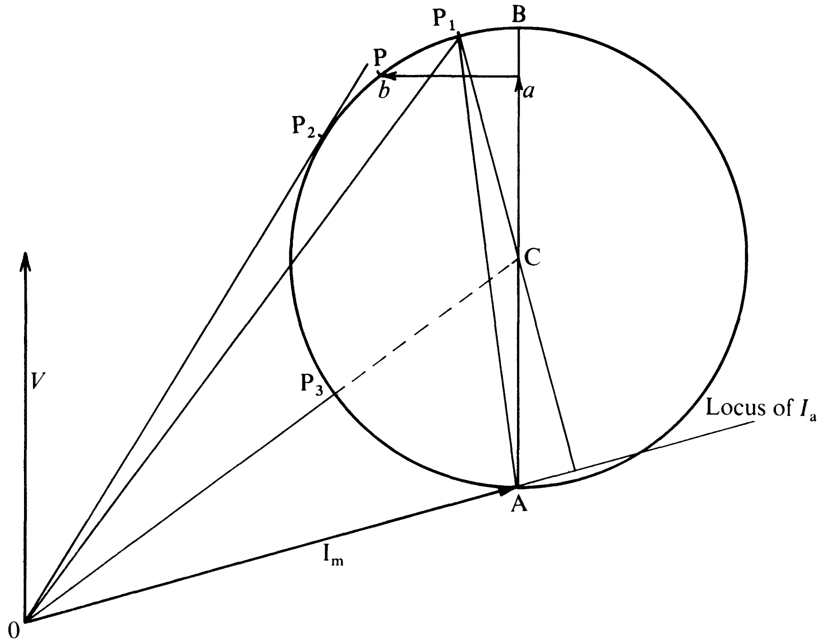


Fig. 6.4. Operating chart for capacitor-start motor.

For a particular operating point P in Fig. 6.4

$$\mathbf{I}_A = \frac{\mathbf{V}}{R_A - j(X_C - X_A)} = a + jb$$

where X_C is the capacitor reactance and X_A is the reactance of the auxiliary phase. Then

$$X_C = X_A + \frac{bR_A}{a} = X_A + \frac{V - aR}{b}. \tag{6.37}$$

The required value of X_C can then be found from the known values of R_A and X_A and the measured values of a and b.

6.3. The shaded-pole motor

The form of construction of the shaded-pole motor is illustrated in Fig. 6.5. The stator is of salient pole construction and carries a single phase winding. Copper rings are placed around one-half of each pole in the manner shown in Fig. 6.5, and the effect of the currents induced in these shading rings is to produce a time lag in the flux under that part of the pole covered by the shading ring relative to the flux under the other part of the pole. This produces the effect of a net rotating flux in the air-gap and the machine will produce a small starting torque. This form of single-phase induction motor has a relatively wide application when only a very low rating is required.

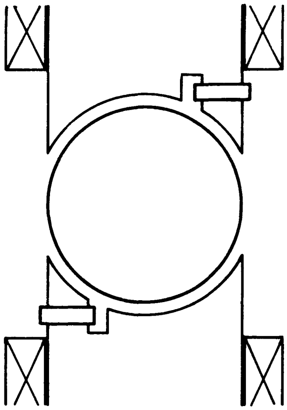


Fig. 6.5. The shaded-pole motor.

6.4. The series commutator motor

The schematic diagram representing the basic form of single-phase series motor is shown in Fig. 6.6. In practice, both the stator and rotor will be laminated and the machine can be operated from both a direct and alternating current system. This motor will have a series characteristic and is used in

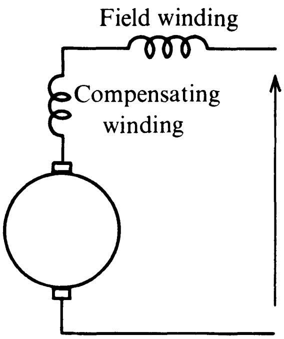


Fig. 6.6. The series commutator motor.

relatively large sizes for traction purposes. However, low frequency operation is necessary because of commutation difficulties, and it is also usual to include a compensating winding connected in series with the armature to compensate for the effects of the armature m.m.f.

The machine current I is given by $I = I_m \sin \omega_s t$, and the flux by $\Phi = \Phi_m \sin \omega_s t$. An expression for the instantaneous torque t_e can be obtained from the torque expression of the d.c. machine given in (3.51) in the form

$$t_e = \frac{PZ}{2\pi a} \Phi_m I_m \sin^2 \omega_s t. \tag{6.38}$$

The average value T_e of this torque is given by

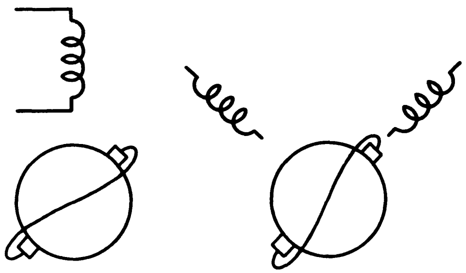
$$T_e = \frac{1}{2} t_e = \frac{PZ}{2\pi a} \frac{\Phi_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \quad \text{or} \quad T_e = \frac{PZ}{2\pi a} \frac{\Phi_m}{\sqrt{2}} I. \tag{6.39}$$

Thus the series motor develops less torque when operating from an a.c. system than when operating from a d.c. system.

This motor has a wide application in fractional horse-power sizes, particularly in domestic appliances, and is then known as the Universal Motor.

6.5. The repulsion motor

Figure 6.7(a) illustrates the form of the repulsion motor for which the short-circuited armature winding is identical to that used for a d.c. machine. The torque produced by the machine is a sine function of the angle α in Fig. 6.7(a), and it follows that the speed of the machine is controlled by the brush position. It can be shown that the arrangement of this machine shown in Fig. 6.7(a) can be simulated by that shown in Fig. 6.7(b) in which the



(a) (b)

Fig. 6.7. The repulsion motor:
(a) Connection diagram. (b) Simulation of connection diagram.

field winding is resolved along and perpendicular to the axis of the armature brushes. Under these conditions, the effect of the winding on the brush axis can be correlated to the effect of the compensating winding on the series motor so that the machine has essentially series characteristics. Maximum torque will be obtained when the brush angle α is approximately equal to 90°e.

In some cases, the action of the repulsion motor is combined with that of the single-phase induction motor to produce the repulsion–induction motor.

The machine is started as a repulsion motor with the corresponding high starting torque and, at some predetermined speed, a centrifugal arrangement short-circuits the commutator so that the machine then operates as a single-phase induction motor.

Reference

1. Jha, C. S., and Daniels, A. R., 'The starting of single-phase induction motors having asymmetrical stator windings in quadrature', *Proc. I.E.E.*, Vol. 106, Part A, no. 28, Aug. 1959.

Tutorial Problems

1. The results of the locked rotor test on a 200 V, 4-pole, 50 Hz, star-connected symmetrical, 3-phase induction motor were as follows: Line voltage 50 V, line current 20 A; total power 866 W. The stator copper loss is 60% of the total copper loss at standstill.

   The machine is connected to a symmetrical 3-phase supply and runs up to speed. If one line is then disconnected from the supply, calculate the slip at which this machine will develop maximum torque. The no-load current can be neglected and an equivalent circuit for a 3-phase machine operating with a single-phase applied voltage can be assumed.

   (*Answer:* 0·103)

2. A single-phase capacitor motor has the following parameters: Rotor resistance referred to main 2 Ω; Rotor reactance referred to main 3 Ω; Stator resistance main 2 Ω, auxiliary 8 Ω; Stator reactance main 3 Ω; auxiliary 4 Ω; Magnetizing reactance referred to main 70 Ω.

   If the turns ratio $k = 1{\cdot}2$, calculate the value of capacitance to be used to produce maximum starting torque and the per-unit value of the corresponding capacitor voltage.

   (*Answer:* 434 μF; 0·676)

3. A symmetrical, 2-phase, 50 Hz, induction motor with a standstill impedance of $10\underline{/46^\circ}$ ohm per phase has one of its phases provided with tappings. The machine is to be operated as a capacitor-start, single-phase motor and is required to operate with the magnitude of the voltage across the capacitor limited to the supply voltage and the magnitude of the auxiliary phase current equal to its balanced 2-phase value. Find the required values of capacitance and turns ratio.

   (*Answer:* 221 μF; 1·44)

4. A symmetrical, 240 V, 50 Hz, 4-pole, 2-phase induction motor is supplied as a single-phase motor fitted with a phase balancer. If the standstill admittance per phase of the machine is $0{\cdot}068\overline{/60^\circ}$ mho, calculate (a) the minimum possible value of negative sequence voltage if the admittance Y is in the form of (i) a capacitor, (ii) a resistor; (b) the value of capacitance to be used to give maximum torque; (c) the value of capacitance to be used to give maximum torque per ampere of line current.

   (*Answer:* 0·236; 0·614; 217 μF; 153 μF)

5. Show that, for a split-phase, single-phase motor, maximum starting torque is developed when the angle between the phase currents is equal to one-half of the standstill phase angle of the main winding.

6. A symmetrical, 2-phase, 50 Hz, induction motor, with a standstill admittance of $0{\cdot}1\overline{/60^\circ}$ mho/phase, is to be operated as a single-phase motor with a phase converter in series with one of its stator phases. The magnitude of the voltage across the phase converter is to be limited to the supply voltage, and the magnitude of the current through the phase converter is to be limited to the phase current of the machine when operating under symmetrical conditions. Find the required component values of the external phase balancer and the magnitude of the neutral current under these conditions expressed as a percentage of the value for balanced 2-phase conditions.

(*Answer:* 5 Ω; 367 μF; 122·4)

7. An induction motor has stator windings in space quadrature and is supplied with a single-phase voltage of 200 V at 50 Hz. The standstill impedance of the main winding is $5{\cdot}2 + j10{\cdot}1$ ohms and the standstill impedance of the auxiliary winding is $19{\cdot}7 + j14{\cdot}2$ ohms. If the external capacitance is to be used to start the machine, sketch the machine stator connections and find, graphically or otherwise, (a) the value of capacitance to be inserted to give the machine maximum starting torque per ampere of starting current; (b) the value of capacitance to be inserted to produce minimum starting current; (c) the value of capacitance to be inserted to give the maximum starting torque.

(*Answer*: 128 μF; 56 μF; 165 μF)

8. A single-phase induction motor of the split-phase type is supplied with 240 V single-phase. The standstill impedance of the main winding is $7{\cdot}71 + j11{\cdot}2$ ohms and the leakage reactance of the auxiliary winding is 29·3 ohms. Determine graphically or otherwise the resistance of the auxiliary winding required to give the machine (a) the maximum starting torque, (b) the maximum starting torque per ampere of starting current.

(*Answer:* 52 Ω; 60·2 Ω)

7. The polyphase synchronous machine

The polyphase synchronous machine can be classified as either a cylindrical rotor or a salient pole machine according to its form of construction. Most turbine-driven synchronous generators are high speed sets with a 2-pole construction and are highly suited to the cylindrical rotor, rotating field type of construction. Hydroelectric power generators are generally low speed multipolar machines of the salient pole type. The synchronous motor can be of either form of construction, although the majority of motors will be of the salient pole type.

In the case of the cylindrical rotor machines, it is assumed that the air-gap between the stator and rotor is constant, so that the reluctance of the mutual magnetic circuits does not depend on the angular position of the rotor. In the case of a salient pole machine, however, there are two definite axes of symmetry and the reluctance will then be a function of the angular position of the rotor. In these circumstances, the techniques used for the derivation of equivalent circuits and performance characteristics of the two types of machine are different.

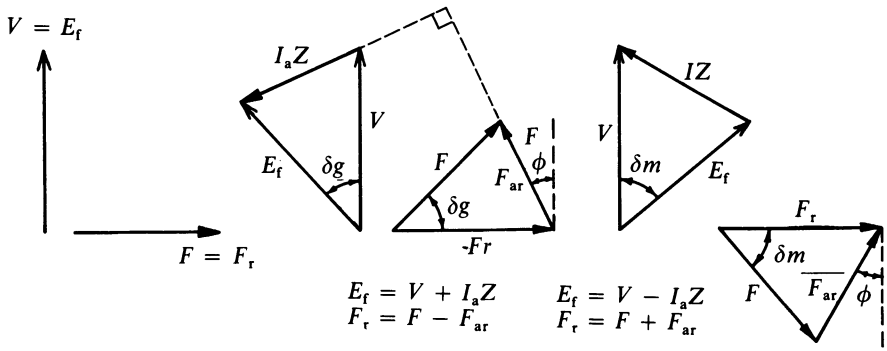
7.1. Basis of operation of cylindrical rotor machines

The basic principles underlying the operation of polyphase synchronous machines have been discussed in chapter 1 where the important concept of load angle has been introduced.

In the basic analysis, it will be assumed that saturation can be neglected and that the m.m.f. set up by the d.c. excitation is sinusoidally distributed in space, so that all alternating voltages and currents are sinusoids. The machine will be considered as a symmetrical polyphase machine and equivalent circuits and phasor diagrams will be drawn on a 'per-phase' basis.

Consider the operation of a 3-phase, rotating field machine connected to a constant voltage, 3-phase system with the machine excitation set to produce an open-circuit voltage equal to the system voltage **V**. Under these conditions, the machine is said to be 'floating' on the system and the corresponding phasor diagram is shown in Fig. 7.1(a) which shows the separate time and space varying quantities. The m.m.f., **F**, produced by the d.c. excitation is equal to the resultant m.m.f., $\mathbf{F}_r$ (which sets up the resultant flux $\mathbf{\Phi}_r$), since the armature current, $\mathbf{I}_a$, and hence the armature reaction m.m.f., $\mathbf{F}_{ar}$, are zero. The internal e.m.f., $\mathbf{E}_f$, shown in Fig. 7.1(a) is set up by the flux produced by the d.c. excitation acting alone and is known as the excitation e.m.f. In the first stage of analysis, the leakage impedance of the armature will be neglected so that the resultant e.m.f., $\mathbf{E}_r$, set up by the resultant flux $\mathbf{\Phi}_r$ will equal the system voltage, $\mathbf{V}$, and will be constant.

When the power applied to the shaft of the machine is increased, with constant excitation, the rotor speed will increase as the machine enters the generating mode. During this transient period, the axis of the field m.m.f. is



(a) (b) (c)

Fig. 7.1. Simplified phasor diagrams:
(a) 'Floating'. (b) Generating. (c) Motoring.

driven ahead, in the direction of rotation, of the axis of the resultant m.m.f. and this condition will continue until the electrical power output from the machine balances the increase in applied shaft power. The axis of the field m.m.f. will then have moved through the necessary load angle δ_g and the final steady state condition is shown in Fig. 7.1(b).

If, from the floating condition, mechanical output power is taken from the shaft, with constant excitation, the rotor speed must decrease as the machine enters the motoring mode. The axis of the field m.m.f. will then lag behind, in the direction of rotation, the axis of the resultant m.m.f. and the transient condition will continue until the electrical input power equals the required value of load power on the shaft. The axis of the field m.m.f. will then have moved through the necessary load angle δ_m and the final steady state condition is shown in Fig. 7.1(c).

In both the above cases, it follows from the phasor diagrams of Fig. 7.1(b) and (c) that

$$F \sin \delta = F_{ar} \cos \phi. \tag{7.1}$$

The armature reaction m.m.f., $\mathbf{F}_{ar}$, is directly proportional to the armature current, $\mathbf{I}_a$, so that, at constant terminal voltage, the quantity $F_{ar} \cos \phi$ is a measure of electrical power. It follows directly from (7.1) that, at constant excitation, electrical power is proportional to the sine of load angle and the power–load angle characteristic of the machine is shown in Fig. 7.2. In the absence of armature resistance, electrical power can be equated to gross mechanical power.

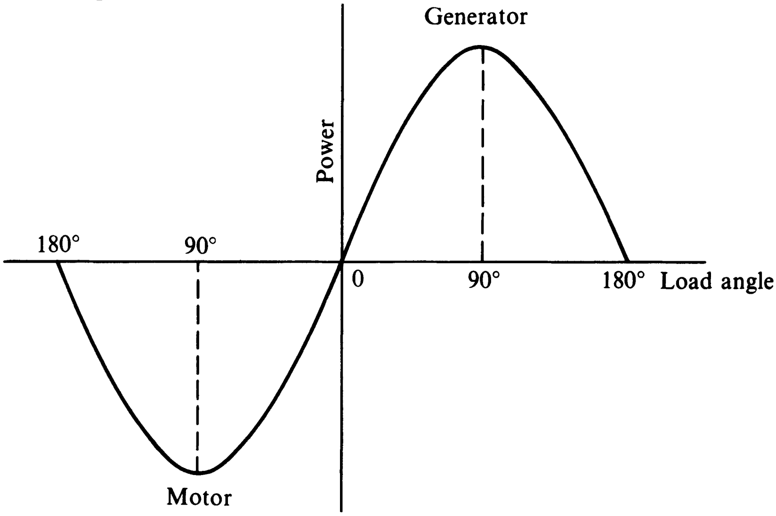


Fig. 7.2. Power–load angle characteristic.

When the machine is operated at constant power, it follows from (7.1) that $F \sin \delta$ must be constant. The loci of $\mathbf{F}$ and $\mathbf{F}_{ar}$ with change in excitation are then as shown in Fig. 7.3. The condition for which the field excitation exceeds

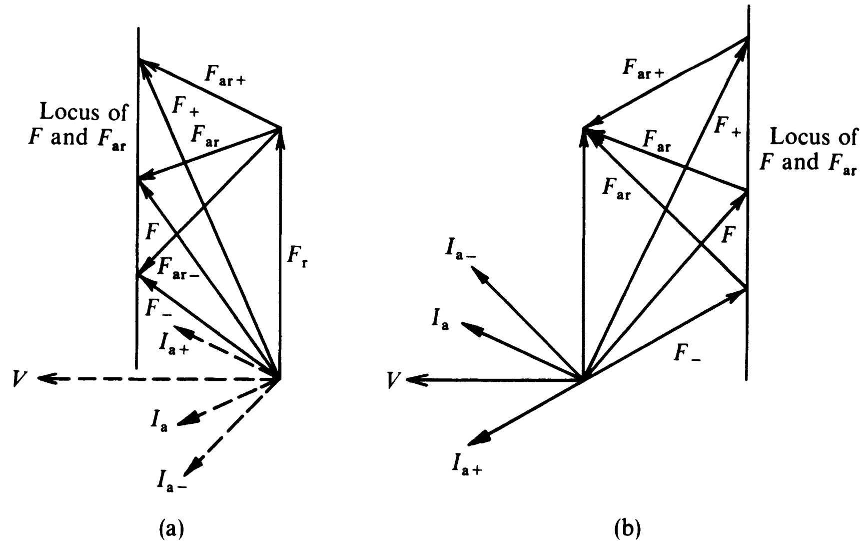


Fig. 7.3. Locus diagram with constant power:
(a) Generating. (b) Motoring.

the value required to set up an excitation e.m.f. equal to the system voltage is known as over-excitation, and the condition for which the field excitation is less than that required to set up an excitation e.m.f. equal to the system voltage is known as under-excitation. It immediately follows from Fig. 7.3(a) that an under-excited generator supplies leading voltamperes reactive (VAr) to the system and an over-excited generator supplies lagging VArs to the system. In exactly the same manner, it follows, from Fig. 7.3(b), that an under-excited motor takes lagging VArs from the system and an over-excited motor takes leading VArs from the system. Thus an over-excited motor operates at leading power-factor and acts as a generator of lagging VArs. In these circumstances, an over-excited motor can be used for power-factor correction in a power system and if the machine operates on no-load, it is known as a synchronous condenser.

7.2. Equivalent circuits for cylindrical rotor machines

In practice, the resultant e.m.f., $\mathbf{E}_r$, produced by the resultant flux, $\boldsymbol{\Phi}_r$, will not equal the terminal voltage $\mathbf{V}$ because of the effects of the armature resistance R_a, and the armature leakage reactance X_1. These quantities are related in the case of a generator by the equation

$$\mathbf{E}_r = \mathbf{V} + \mathbf{I}_a(R_a + jX_l). \tag{7.2}$$

The resultant e.m.f. will differ from the excitation e.m.f. because of the effects of armature reaction and, in the unsaturated case, the generator phasor diagram can be drawn as shown in Fig. 7.4 in which the e.m.f. $\mathbf{E}_{ar}$

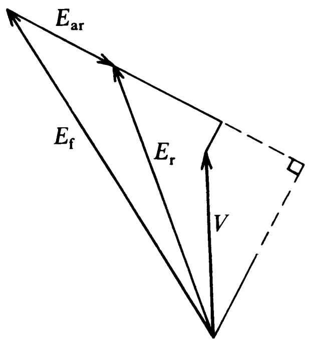


Fig. 7.4. Generator phasor diagram.

represents the effect of armature reaction and lags the current $\mathbf{I}_a$ by 90°. The resultant e.m.f. can then be written as

$$\mathbf{E}_r = \mathbf{E}_f + \mathbf{E}_{ar} = \mathbf{E}_f - j\mathbf{I}_a X_{ar} \tag{7.3}$$

where X_{ar} is known as the reactance of armature reaction.

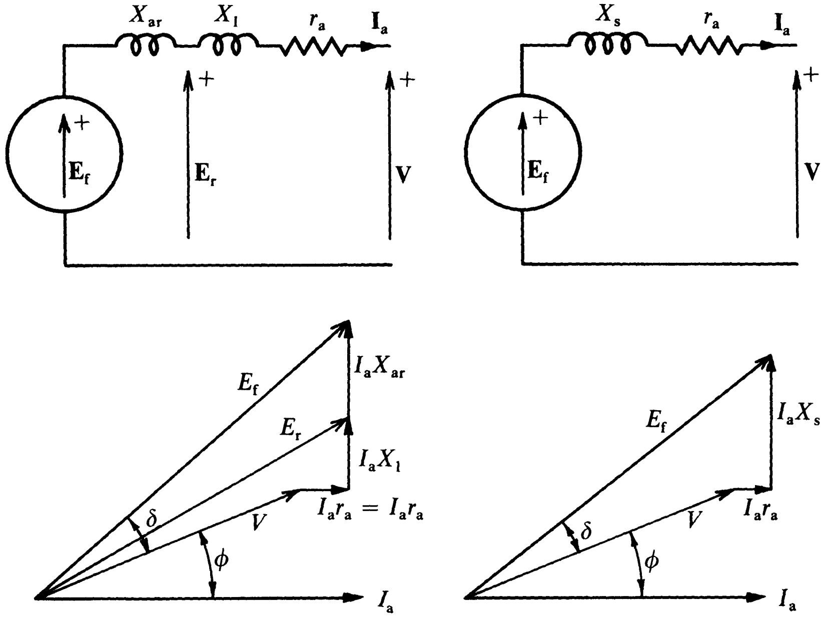


Fig. 7.5. Generator equivalent circuit and phasor diagram.

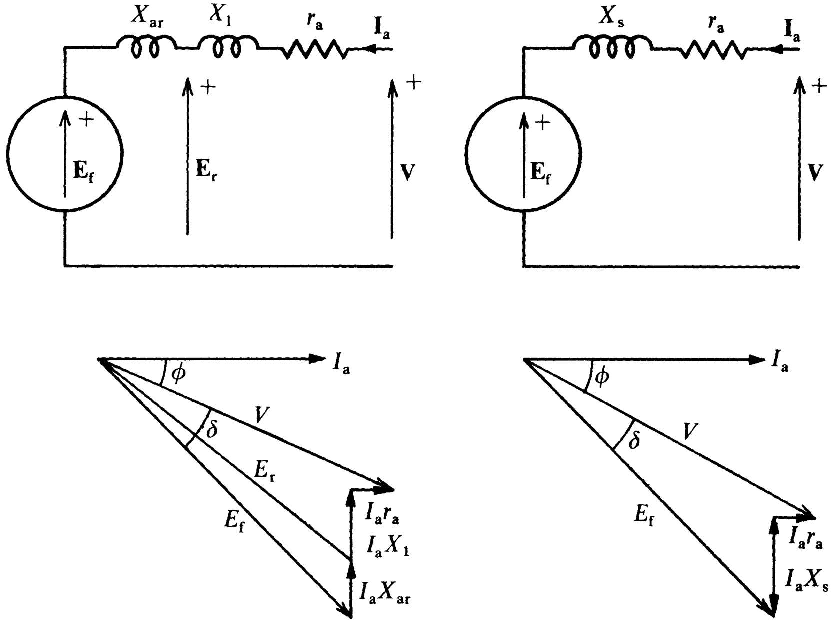


Fig. 7.6. Motor equivalent circuit and phasor diagram.

When the value of $\mathbf{E}_r$ from (7.3) is substituted in (7.2) it follows that

$$\mathbf{E}_f = \mathbf{V} + \mathbf{I}_a[R_a + j(X_l + X_{ar})] \quad \text{or} \quad \mathbf{E}_f = \mathbf{V} + \mathbf{I}_a(R_a + jX_s) \quad (7.4)$$

where $X_s = X_l + X_{ar}$ is known as the synchronous reactance and $\mathbf{Z}_s = R_a + jX_s$ is known as the synchronous impedance.

Forms of equivalent circuit and corresponding phasor diagrams for a synchronous generator are shown in Fig. 7.5, and the corresponding equivalent circuits and phasor diagrams for a synchronous motor are shown in Fig. 7.6.

Performance calculations under steady-state conditions will normally be based on the equivalent circuits of Figs. 7.5 and 7.6, and unsaturated values for the synchronous reactance can be found from the measured value of armature resistance and the results of two simple tests.

7.3. Open-circuit and short-circuit characteristics

The open-circuit characteristic (O.C.C.) is the relationship between the voltage appearing at the machine terminals on open-circuit and the field current I_f (or m.m.f. F) when the machine is driven at synchronous speed. Corresponding values of open-circuit voltage and field current are measured for a range of voltages up to approximately 130% of rated voltage and plotted in the manner shown in Fig. 7.7, which takes the form of a normal magnetization curve. The extension of the initial linear part of the O.C.C. is known as the air-gap line.

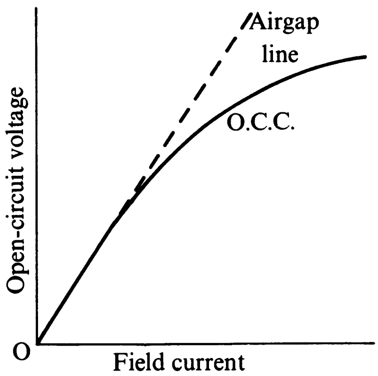


Fig. 7.7. Open-circuit characteristic.

The short-circuit characteristic (S.C.C.) is the relationship between the armature current I_a and the field current I_f (or m.m.f. F), when the machine is driven at synchronous speed with a symmetrical short-circuit applied to its armature terminals, and corresponding values would normally be measured for a range of armature currents up to 150% of the rated value. The form of the S.C.C. is shown in Fig. 7.8 and will usually be a straight line through the origin.

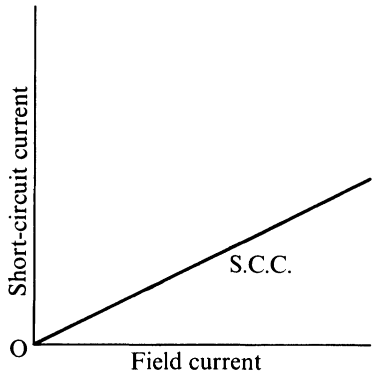


Fig. 7.8. Short-circuit characteristic.

A value for the synchronous impedance per phase of the machine can be obtained directly from these two characteristics which are shown plotted on the same axes in Fig. 7.9. In this figure, for the excitation OA, the unsaturated value of synchronous impedance is given by AD/AB and the so-called 'saturated' value by AC/AB.

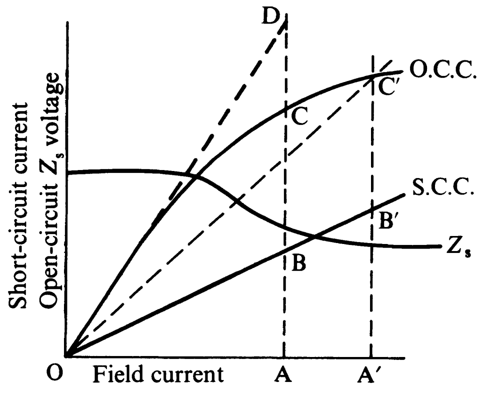


Fig. 7.9. Combined O.C. and S.C. characteristics.

It is, however, difficult in practice to consider the synchronous impedance as a variable quantity and for operation near rated voltage, a value for the synchronous impedance can be obtained by assuming that the machine is equivalent to an unsaturated machine with an air-gap line passing through the rated voltage point as shown in Fig. 7.9. Under these conditions, the synchronous impedance is given by A^1C^1/A^1B^1.

7.4. Basis of operation of salient pole machines

It has previously been noted that the air-gap of the salient pole machine is not uniform and will, therefore, present two definite axes of geometric symmetry. In these circumstances, it is usual to use the two-reaction theory of Blondel and to resolve along two axes at right-angles, the so-called direct axis (d-axis) corresponding to the axis of a field pole and the quadrature axis (q-axis) corresponding to the axis of the interpolar space in the manner shown in Fig. 7.10.

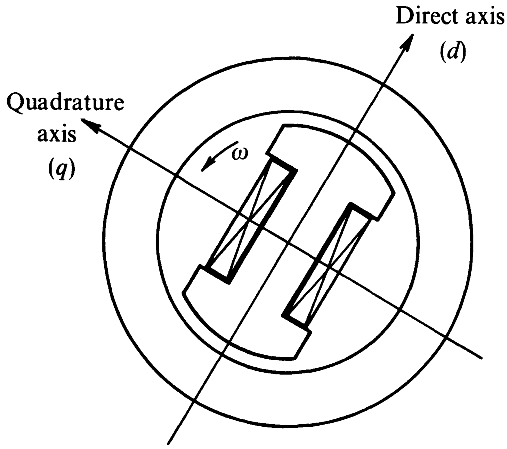


Fig. 7.10. Salient pole machine.

Consider the case of an unsaturated salient pole machine. The armature reaction m.m.f. can be resolved into components along the direct and quadrature axes and, since the geometric configurations along these two axes are different, it follows that the component fluxes produced on these axes by the relevant m.m.f.s will be different. It is apparent that, since the reluctance on the quadrature axis will be greater than that on the direct axis, the armature reaction reactance X_{aq} associated with the quadrature component of armature current will be less than the armature reaction reactance X_{ad} associated with

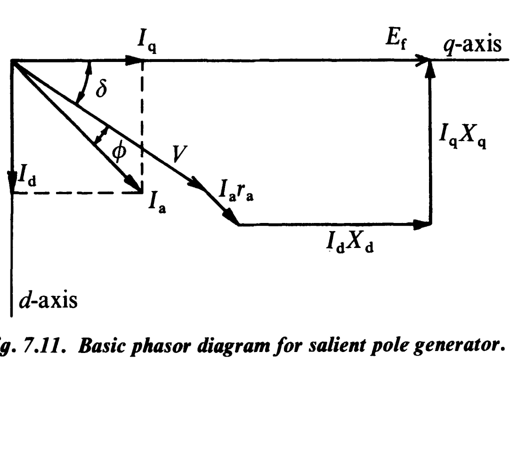


Fig. 7.11. Basic phasor diagram for salient pole generator.

the direct component of armature current. It follows directly that the synchronous reactance X_d on the direct axis will be greater than the synchronous reactance X_q on the quadrature axis, since the armature leakage reactance X_l will be the same on both axes.

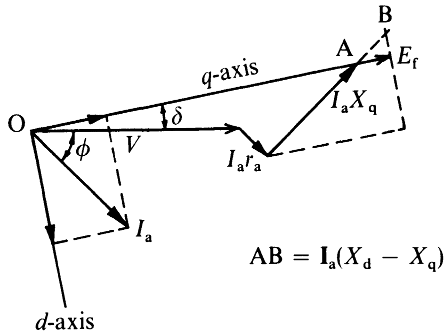


Fig. 7.12. Complete generator phasor diagram.

The basic phasor diagram of a salient pole generator is shown in Fig. 7.11, in which it is assumed that the angle between the excitation e.m.f. $\mathbf{E}_f$ and the armature current $\mathbf{I}_a$ is known. The excitation e.m.f. can be written, from Fig. 7.11, in the form

$$\mathbf{E}_f = \mathbf{V} + \mathbf{I}_a R_a + jX_d\mathbf{I}_d + j\mathbf{I}_q X_q. \quad (7.5)$$

Now $\mathbf{I}_a = (\mathbf{I}_d + \mathbf{I}_q)$ so that

$$j\mathbf{I}_a X_q = j\mathbf{I}_d X_q + j\mathbf{I}_q X_q. \quad (7.6)$$

When the value of $j\mathbf{I}_q X_q$ from (7.6) is substituted in (7.5), $\mathbf{E}_f$ can be written

$$\mathbf{E}_f = \mathbf{V} + \mathbf{I}_a R_a + j\mathbf{I}_a X_q + j\mathbf{I}_d(X_d - X_q). \quad (7.7)$$

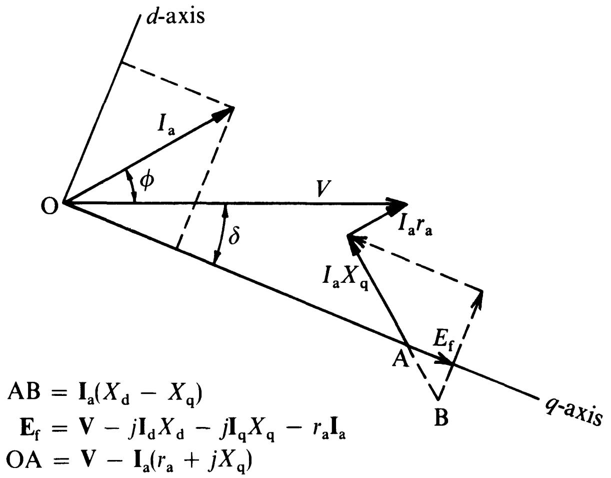


Fig. 7.13. Complete motor phasor diagram.

In practice, the values of terminal voltage **V** and armature current $\mathbf{I}_a$ will be known and the corresponding phasor diagram can be drawn by the construction shown in Fig. 7.12 which is based on the form of (7.7). The phasors representing **V** and $\mathbf{I}_a$ can be drawn and the quantity $\mathbf{V} + \mathbf{I}_a(R_a + jX_q)$ set up corresponding to the phasor OA in Fig. 7.12, which fixes the direction of the quadrature axis as OA. The direct and quadrature axis components of $\mathbf{I}_a$ can then be found and the complete phasor diagram can be drawn. The corresponding phasor diagram for a salient pole motor is shown in Fig. 7.13.

Simple equivalent circuits representing the operation of salient pole machines cannot be drawn unless the direct- and quadrature-axis circuits are assumed to have separate existences, and performance calculations will normally be based on the use of phasor diagrams and the known values of direct- and quadrature-axis synchronous reactance.

7.5. Determination of synchronous reactances

The value of the unsaturated direct-axis synchronous reactance can be obtained by the method described in section 7.3, and allowance for saturation made according to the methods of reference 1. In order to obtain a value for the quadrature-axis synchronous reactance, additional experimental information is required. One such method is known as the *slip test* in which balanced voltages of rated frequency are applied to the armature of the machine with the field on open-circuit and the rotor is driven at a small slip with respect to the armature rotating m.m.f. Oscillograms of armature voltage and current and field voltage are taken and are of the form shown in

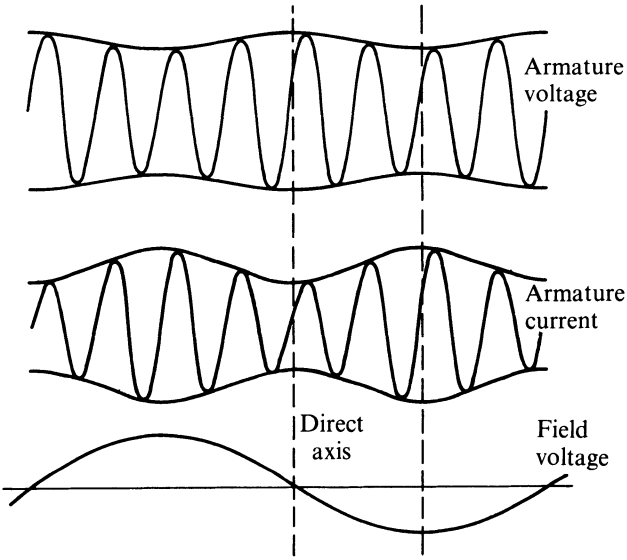


Fig. 7.14. Results of slip test.

Fig. 7.14. The direct-axis synchronous impedance is then the maximum value of the ratio of armature voltage to current and the quadrature-axis synchronous impedance is the minimum value of this ratio. It is normal to obtain the ratio of these impedances from this test and to use the known value of the direct-axis value to find the quadrature-axis value. In order to obtain reasonable accuracy, the value of slip used during this test should be small and, under these conditions, the machine will tend to synchronize and operate as a reluctance motor. The machine must therefore be operated at reduced voltage and the synchronous reactances obtained will be unsaturated values.

An alternative method of obtaining a value for the quadrature-axis synchronous impedance is the *maximum lagging current test*, for which the machine is operated as a synchronous motor on no-load. The excitation is reduced to zero, then reversed and increased until the stability limit is reached. It can be shown that, under these conditions, the quadrature-axis synchronous impedance is the ratio of armature phase voltage to current and this value of reactance will be a saturated value.

7.6. Voltage regulation of generators

Voltage regulation is defined as the percentage rise in terminal voltage when the load is removed. That is,

$$\text{Regulation} = \frac{E_f - V}{V} \times 100\%. \tag{7.8}$$

One of the simplest ways of obtaining voltage regulation is to use the equivalent circuit and phasor diagram of Fig. 7.5 in conjunction with the open-circuit and short-circuit characteristics of Fig. 7.9. The value of synchronous impedance can be obtained directly from the O.C.C. and S.C.C. using either the air-gap line or making some allowance for saturation as previously noted in reference 1, and the value of E_f for specified load conditions obtained directly. This method is known as the e.m.f. method but suffers from the disadvantage that, for short-circuit, saturation is low. Hence the synchronous impedance and regulation will be high and the method is known as a 'pessimistic' method.

An alternative method of obtaining voltage regulation from the O.C.C. and S.C.C. is known as the m.m.f. method in which it is assumed that rated voltage corresponds to the resultant m.m.f. on the O.C.C. Then the resultant m.m.f. F_r is set up from the O.C.C., and the armature reaction m.m.f. F_{ar} corresponding to the load current is set up from the S.C.C.

The excitation m.m.f. F is obtained from the relationship,

$$\mathbf{F} = \mathbf{F}_r + \mathbf{F}_{ar} \tag{7.9}$$

and the value of E_f is obtained on the O.C.C. corresponding to the m.m.f. F. This method is, however, approximate because it assumes that the leakage impedance is zero. The value of armature reaction m.m.f. $\mathbf{F}_{ar}$ is again obtained from the S.C.C. and will therefore be low. In these circumstances, the value of excitation e.m.f. and hence regulation will be low and the method is known as an 'optimistic' method.

7.7. Paralleling of polyphase synchronous machines

Most synchronous machines will operate in parallel with other synchronous machines and the process of connecting one machine to another or to an infinite busbar system is known as synchronizing.

In order that two 3-phase voltage sources can be successfully connected in parallel, it is necessary that the system voltages are of the same magnitude and frequency, of the same phase-sequence and that the respective phase voltages are in phase at the instant of switching. The magnitudes of the system voltages can be found using a voltmeter and their phase sequences found using a phase-sequence meter (ref. 2). The conditions governing frequency and instantaneous phasing can be satisfied using a synchroscope.

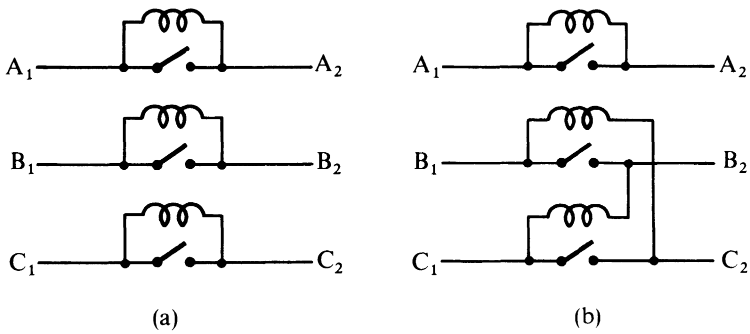


Fig. 7.15. Three-lamp synchroscope:
(a) Straight connection. (b) Cross connection.

The most elementary form of synchroscope consists of three lamps connected across the paralleling switch in either of the two manners shown in Fig. 7.15. If the lamps are connected in the manner shown in Fig. 7.15(a), the instantaneous voltages across all three lamps will be the same and will be zero when the two systems are in phase. If two of the lamps are cross-connected in the manner shown in Fig. 7.15(b), the instantaneous voltages across the three lamps will be different, and the lamps will be illuminated in cyclic order. The paralleling switch should be closed at the instant when the straight connected lamp A is out and the other two lamps are at equal brilliancy. The rate at which the lamps flash depends on the frequency difference between the two systems. The order in which the lamps flash is determined by the relative speed between the two systems and this method shows which of the frequencies is the greater.

In most practical systems, a more elegant indicator in the form of a small 2-pole motor (ref. 3) would be used as a synchroscope.

7.8. Cylindrical rotor machines on infinite busbars

Consider the operation of a 3-phase generator connected to a constant voltage, constant frequency system. The phasor diagram corresponding to this condition is shown in Fig. 7.16 in which the system voltage **V** is taken as

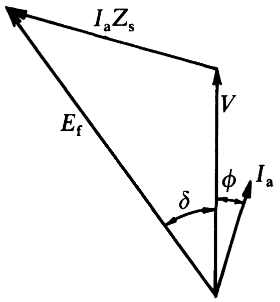


Fig. 7.16. Generator on infinite busbars.

reference. The nett electrical output power P_o from the generator is given by

$$P_o = 3\mathscr{R}\mathbf{V}\mathbf{I}_a^* \tag{7.10}$$

where $\mathscr{R}$ is the 'real part of' and $\mathbf{I}_a^*$ is the conjugate of $\mathbf{I}_a$. It follows from (7.10) that

$$\mathbf{I}_a = \frac{\mathbf{E}_f - \mathbf{V}}{\mathbf{Z}_s} = \frac{E_f\underline{|\delta} - V}{Z_s\underline{|\theta}}.$$

Thus

$$\mathbf{I}_a^* = \frac{E_f\overline{\backslash\delta} - V}{Z_s\overline{\backslash\theta}} = \frac{E_f\overline{\backslash\delta - \theta} - V\underline{|\theta}}{Z_s}. \tag{7.11}$$

The value of $\mathbf{I}_a^*$ from (7.11) can be substituted in (7.10) to give

$$P_o = 3\mathscr{R}V\left[\frac{E_f\overline{\backslash\delta - \theta} - V\underline{|\theta}}{Z_s}\right] \quad \text{or} \quad P_o = \frac{3V}{Z_s}\left[E_f\cos(\delta - \theta) - V\cos\theta\right]. \tag{7.12}$$

If the machine is operated at constant excitation, E_f will be constant and it can be seen from (7.12) that maximum electrical output power P_{om} is obtained when the load angle δ is given by

$$\delta = \theta = \tan^{-1}\frac{X_s}{R_a}. \tag{7.13}$$

Then, from (7.12)

$$P_{om} = \frac{3V}{Z_s}(E_f - V\cos\theta). \tag{7.14}$$

It follows, from (7.14), that, theoretically, the power available from a generator increases indefinitely with excitation. In practice, however, there will be a maximum permissible value of field current and a power limit imposed by the capacity of the prime mover.

If armature resistance is neglected, $Z_s\angle\theta \equiv X_s\angle 90^\circ$ and (7.12) can be written as

$$P_o = \frac{3VE_f\sin\delta}{X_s}. \tag{7.15}$$

Under these conditions, the electrical output power will be at maximum when the load angle δ is 90°.

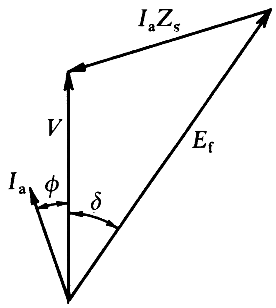


Fig. 7.17. Motor on infinite busbars.

The corresponding phasor diagram for a motor operating on infinite busbars is shown in Fig. 7.17, and the gross mechanical output power P_g from the motor is given by

$$P_g = 3\mathcal{R}\mathbf{E}_f\mathbf{I}_a^*. \tag{7.16}$$

It follows from Fig. 7.17 that

$$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{\mathbf{Z}_s} = \frac{V - E_f\angle{-\delta}}{Z_s\angle\theta}.$$

Then

$$\mathbf{I}_a^* = \frac{V - E_f\angle\delta}{Z_s\angle{-\theta}} = \frac{V\angle\theta - E_f\angle\delta + \theta}{Z_s}. \tag{7.17}$$

The value of $\mathbf{I}_a^*$ from (7.17) can be substituted in (7.16) to give

$$P_g = 3\mathcal{R}E_f\angle{-\delta}\left[\frac{V\angle\theta - E_f\angle\delta + \theta}{Z_s}\right]$$

or

$$P_g = \frac{3E_f}{Z_s}[V\cos(\delta - \theta) - E_f\cos\theta]. \tag{7.18}$$

If the machine is operated at constant excitation, E_f will be constant and it can be seen from (7.18) that maximum gross output power P_{gm} will occur when the load angle δ is given by

$$\delta = \theta = \tan^{-1}\frac{X_s}{R_a}. \tag{7.19}$$

Then, from (7.18),

$$P_{gm} = \frac{3E_f}{Z_s}[V - E_f \cos\theta]. \tag{7.20}$$

When (7.20) is differentiated with respect to E_f it follows that, theoretically, ultimate mechanical output power is obtained from a synchronous motor when E_f is given by

$$E_f = \frac{V}{2\cos\theta} = \frac{VZ_s}{2R_a}. \tag{7.21}$$

In practice, there will be a maximum permissible value of excitation.

When armature resistance is neglected, (7.18) can be written as

$$P_g = \frac{3E_f V \sin\delta}{X_s}. \tag{7.22}$$

Under these conditions, the gross mechanical output power will be a maximum when the load angle δ is 90°.

It can be seen from (7.15) and (7.22) that the power-load angle characteristic for a cylindrical rotor machine is a sinusoid with its maximum value depending on the excitation.

Consider a cylindrical rotor machine with negligible armature resistance

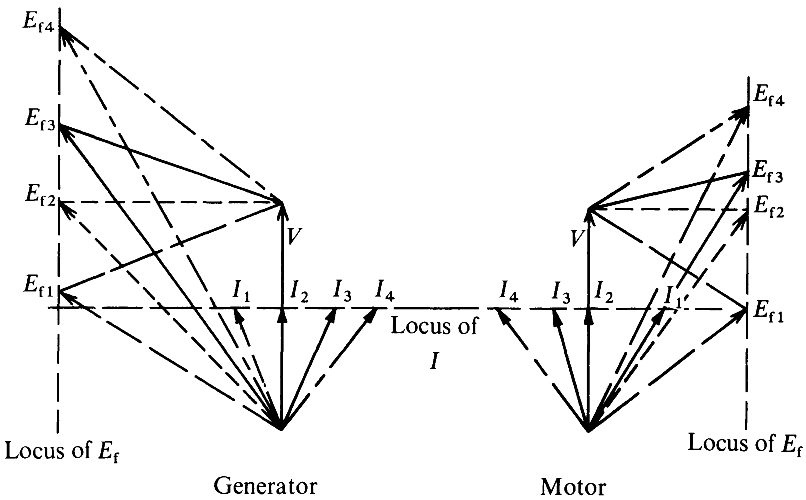


Fig. 7.18. Operation with fixed power.

connected to infinite busbars and operating with constant power. Since the nett electrical power P_0 equals the gross mechanical power P_g, it follows from (7.15) and (7.22) that

$$P_o = P_g = \frac{3E_f V \sin \delta}{X_s} = 3VI \cos \phi. \tag{7.23}$$

Thus $E_f \sin \delta$ and $I_a \cos \phi$ are both constant with constant power and the loci of $\mathbf{E}_f$ and $\mathbf{I}_a$ for differing values of excitation in both the motoring and generating cases are shown in Fig. 7.18. The relationship between the armature current and the field current under these conditions is illustrated in Fig. 7.19, and these curves form the characteristic Vee curves. Lines of constant power-factor are also shown on Fig. 7.19, and it is apparent that the point of

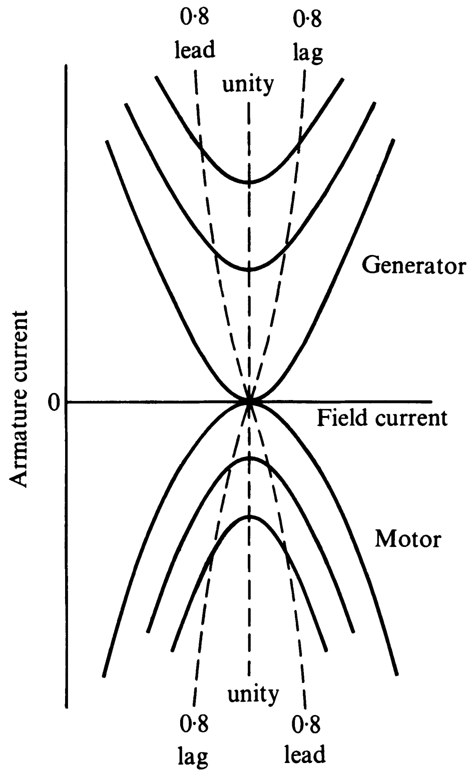


Fig. 7.19. Vee curves.

minimum armature current corresponds to the unity power-factor point. The curves shown in Fig. 7.19 are typical experimental curves and the effect of armature resistance and saturation is to move the locus of the unity power-factor line from a theoretical vertical line as shown.

7.9. Salient pole machines on infinite busbars

The phasor diagram corresponding to the operation of a 3-phase, salient pole, synchronous generator with negligible armature resistance on infinite busbars is shown in Fig. 7.20. The net electrical output power P_o is the sum

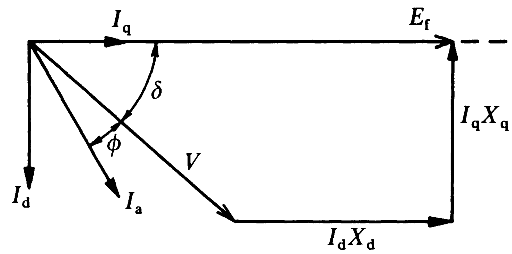


Fig. 7.20. Salient pole generator on infinite busbars.

of the products of the in-phase components of voltage and current and can be written as

$$P_o = 3[(V \sin \delta)I_d + (V \cos \delta)I_q]. \tag{7.24}$$

Then, from Fig. 7.20,

$$I_dX_d = E_f - V \cos \delta \tag{7.25}$$

$$I_qX_q = V \sin \delta. \tag{7.26}$$

When the value of I_d from (7.25) and the value of I_q from (7.26) are substituted in (7.24), P_o can be written

$$P_o = \frac{V \sin \delta}{X_d}[E_f - V \cos \delta] + \frac{V \cos \delta}{X_q}(V \sin \delta)$$

$$= \frac{E_f V \sin \delta}{X_d} - \frac{V^2 \sin \delta \cos \delta}{X_d} + \frac{V^2 \sin \delta \cos \delta}{X_q}$$

That is, $$P_o = \frac{E_f V \sin \delta}{X_d} + \frac{V^2(X_d - X_q)}{2X_dX_q} \sin 2\delta. \tag{7.27}$$

The term $\frac{V^2(X_d - X_q)}{2X_dX_q} \sin 2\delta$ in (7.27) is the reluctance torque introduced by the effects of the salient poles and is independent of the excitation.

Under the condition of negligible armature resistance, the gross mechanical output from a salient pole motor will equal the net electrical power from a generator. The power–load angle characteristic for a salient pole machine is shown in Fig. 7.21. For a specified operating voltage and fixed direct-axis synchronous reactance, the salient pole machine will operate at a lower value of load angle than a cylindrical rotor machine for a given power.

The condition for a maximum power at fixed excitation can be found by differentiating (7.27) with respect to load angle δ. Then

$$\frac{dP_o}{d\delta} = \frac{E_f V \cos\delta}{X_d} + \frac{V^2(X_d - X_q)}{X_d X_q}\cos 2\delta = 0$$

that is,
$$E_f \cos\delta + V\left(\frac{X_d - X_q}{X_q}\right)(2\cos^2\delta - 1) = 0.$$

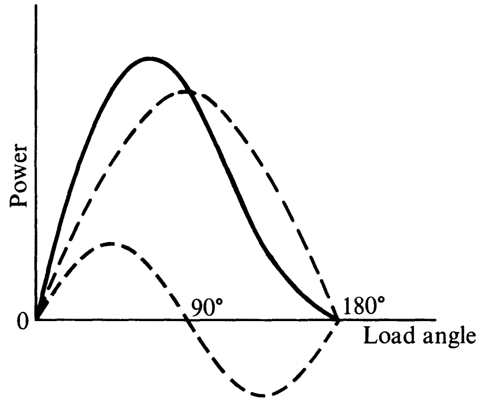


Fig. 7.21. Power–load angle characteristic.

On solution, the load angle for maximum power is given by

$$\cos\delta = \frac{-E_f \pm \left[E_f^2 + 8V^2\left(\frac{X_d - X_q}{X_q}\right)^2\right]^{\frac{1}{2}}}{4V\left(\frac{X_d - X_q}{X_q}\right)}. \qquad (7.28)$$

When the salient pole machine is analysed as a cylindrical rotor machine, by assuming that $X_d = X_q$, calculated Vee curves are reasonably accurate but large errors are involved in the values of load angle.

7.10. Parallel operation of generators

Most large power distribution systems consist of a number of generators in parallel supplying a common busbar system such that the system capacity is assumed to be infinite compared with the rating of an individual machine. In these circumstances, it is usual to assume that the synchronous impedance of the machine is constant under all loading conditions and, in many cases, armature resistance will be neglected. The output power from a generator operating under these conditions will be controlled solely by the power from the prime mover, and the division of load between the generators can only be changed by control of their respective prime movers. The operating power-factor of a machine is governed directly by the excitation current.

Consider two identical machines A and B operating in parallel and supplying a common load of fixed impedance. The phasor diagram corresponding to the condition for which the machines share the load equally and operate at the same power-factor is shown in Fig. 7.22(a). If the excitation of machine A is now decreased, the excitation of machine B must be increased in the manner shown in Fig. 7.22(b), if the terminal voltage is to be maintained constant.

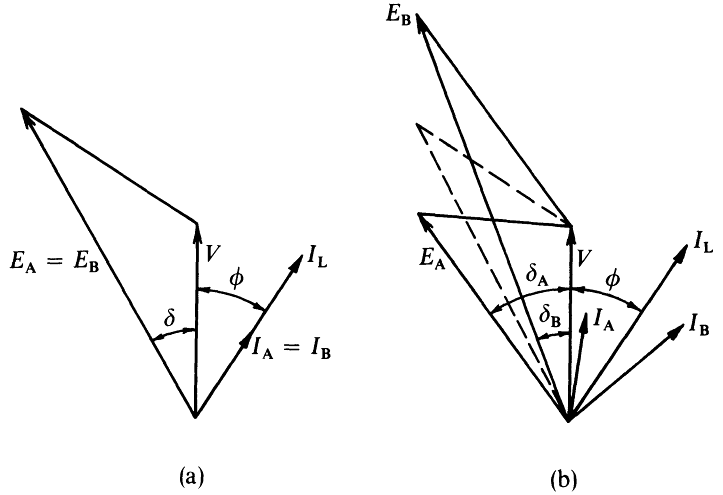


Fig. 7.22. Generators in parallel.

If the excitations are not controlled in this manner, the terminal voltage must change and produce a change in the total load. In these circumstances, since the power from both prime movers has not been changed, the speed of the machines and hence the supply frequency must change. It is usual, in practice, to control the throttle setting of the prime mover to obtain constant system frequency and to ensure that the load is divided between the machines in proportion to their ratings. Automatic voltage regulators are used in the excitation circuit to control the operating power-factor of the machine.

The performance of a synchronous generator supplying power to a large capacity system will normally be obtained from a locus diagram designed to show the simultaneous variations of the necessary quantities.

7.11. Generator operating charts

Consider the phasor diagram shown in Fig. 7.23 for a cylindrical rotor machine with negligible armature resistance. For fixed excitation, the length OC is fixed and the locus of $\mathbf{E}_f$ will be a circle, centre O, radius OC. The machine will be assumed to be connected to constant voltage busbars, so that the length OA in Fig. 7.23 is fixed and the points O and A are fixed. Then, from Fig. 7.23,

$$AM = I_a X_s \cos\phi \quad \text{and} \quad AN = I_a X_s \sin\phi.$$

Hence the length AM is proportional to output power and the length AN is proportional to output volt.amp-reactive. A simplified form of operating

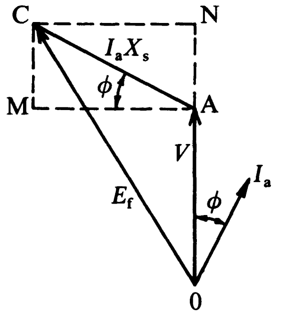


Fig. 7.23. Phasor diagram for cylindrical rotor generator.

chart for a 3-phase, star-connected, cylindrical rotor machine is shown in Fig. 7.24, such that

$$AC = 3VI_a = (I_a X_s)\frac{3V}{X_s} \quad \text{and} \quad OA = \frac{(\sqrt{3}\,V)^2}{X_s}.$$

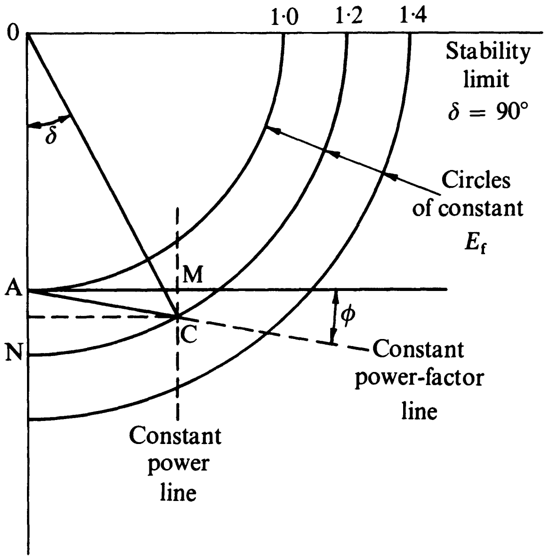


Fig. 7.24. Simplified operating chart.

Now
$$X_s = X_{s\,pu}\frac{V_r^2}{(VA)_r}$$

and it follows that
$$OA = \frac{(\sqrt{3}\,V)^2}{V_r^2}\frac{(VA)_r}{X_{s\,pu}} = \frac{(VA)_r}{X_{s\,pu}} \quad \text{when} \quad \frac{\sqrt{3}\,V}{V_r} = 1$$

where V_r is the rated voltage and $(VA)_r$ is the rated volt-amperes.

The length OA in Fig. 7.24 is proportional to the excitation e.m.f. and determines the scale of constant e.m.f. circles. The prime mover will impose a limit on the available power, and then there will also be a maximum per-

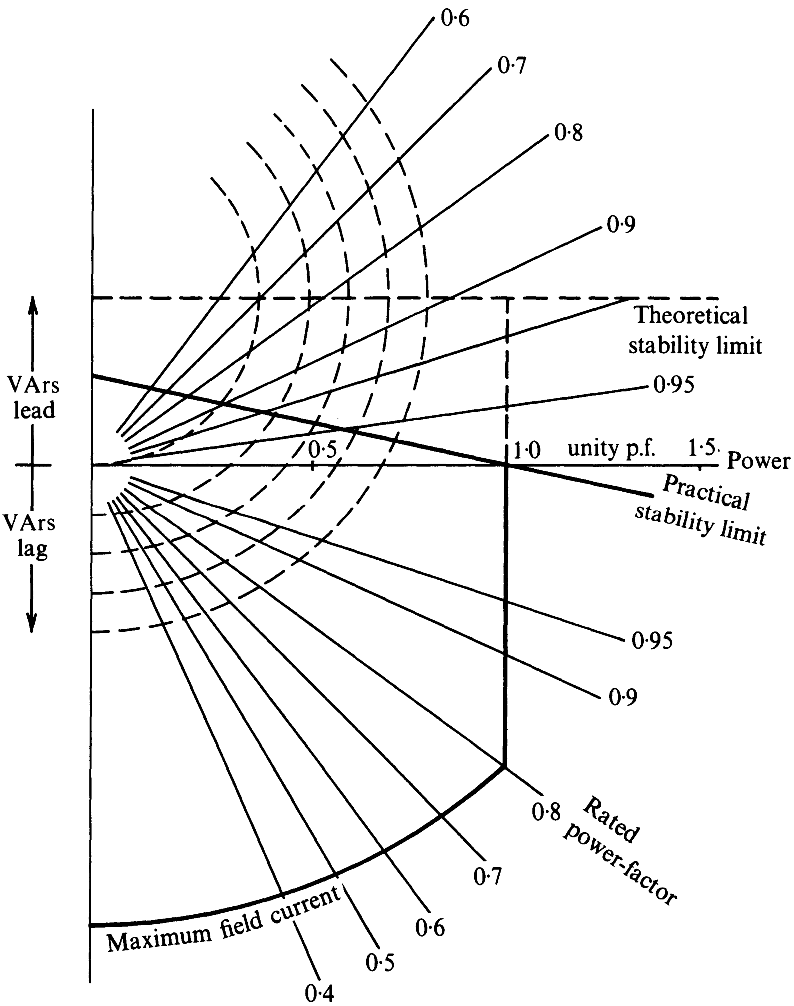


Fig. 7.25. Practical operating chart.

missible value of field current imposing an excitation limit and a maximum permissible value of armature current. The theoretical limit of stability occurs when $\delta = 90^\circ$ and it is apparent that there must be a safety margin between the theoretical limit and that used in practice. The practical stability limit can be expressed as a fixed value of leading volt.amp-reactive or as a 10% stability margin on excitation. The form of the practical operating chart for a cylindrical rotor machine is shown in Fig. 7.25.

7.12. Starting of synchronous motors

It has previously been noted that the synchronous motor does not produce a steady starting torque and it is therefore necessary to provide a starting method for this motor.

A small induction motor, a so-called Pony motor, can be direct-coupled to the main motor and used to run the synchronous motor up to a speed just below the synchronous speed. If d.c. excitation is then applied, the machine will pull into step and operate as a synchronous motor.

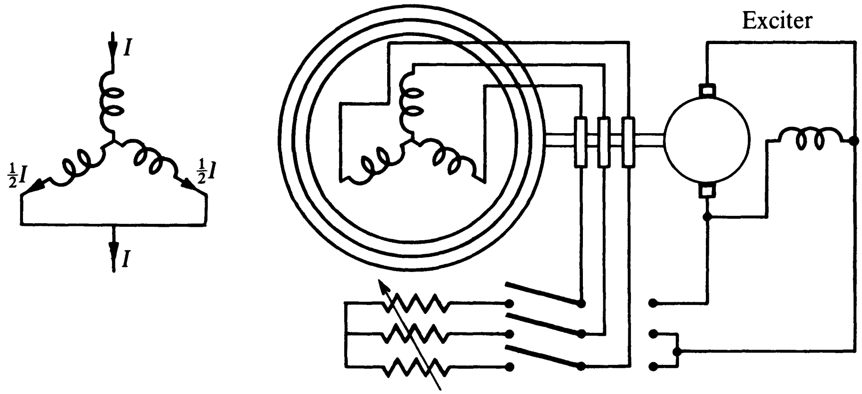


Fig. 7.26. Synchronous-induction motor.

It is usual to provide all synchronous machines with short-circuited damper windings on the field circuit, and these windings can be used to produce an asynchronous torque when the armature is supplied with polyphase voltage. Thus the machine can be run up to just below synchronous speed with the field circuit shorted through a resistor.

The most elegant starting method for a synchronous motor is to use a machine in the form of the so-called Synchronous Induction motor. The field winding is in the form of a standard 3-phase winding and the machine is started as a 3-phase wound-rotor induction motor with a variable external rotor resistance. When the starting resistor has been short-circuited the rotor is reconnected in the manner shown in Fig. 7.26. Alternatively, the exciter can be permanently in circuit.

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Tutorial Problems

1. A 100 h.p., 415 V, 3-phase, star-connected synchronous motor with a synchronous reactance of 2 ohms per phase and negligible armature resistance has a full load efficiency of 0·92. Calculate the excitation e.m.f., the total power developed and the operating load angle for full-load at power-factor 0·8 lead and power-factor 0·8 lag.
(*Answer:* 81·1 kW; 142 A; 466 V; 29°; 240 V; 73°)

2. A 6·6 kV, 50 Hz, star-connected, 3-phase synchronous generator having a synchronous reactance of 9·5 ohms per phase operates on 6·6 kV infinite busbars with the field current set to produce an excitation e.m.f. of 1·1 per-unit. Calculate (i) the maximum value of nett power the machine can deliver to the busbars, its operating power-factor and (ii) the total power developed under these conditions.
(*Answer:* 5·05 MW; 0·74 lead; 5·05 MW)

3. Repeat Problem No. 2 for a machine with a synchronous impedance of $1{\cdot}16 + j9{\cdot}43$ ohms per phase.
(*Answer:* 4·49 MW; 0·673 lead; 4·85 MW)

4. A 3-phase, star-connected turbo-alternator with a synchronous reactance of 10 ohms per phase supplies 220 A at unity power-factor to 11,000 V constant frequency busbars. If the throttle is held constant while the excitation e.m.f. is increased by 25% determine the new current and power-factor. Neglect armature resistance.
(*Answer:* 281 A; 0·806 lead)

5. A 6·6 kV, 3-phase, 50 Hz, star-connected synchronous motor has a synchronous impedance of $12 \exp j82°$ ohms per phase. The field current of the machine is set to produce a per-unit excitation e.m.f. of 1·3. The machines operate on constant frequency 6·6 kV infinite busbars. Calculate the operating power-factor and the line current of the machine when the input power is 800 kW. What is the maximum value of gross output power the machine can develop with this value of field current? A graphical, or part-graphical, solution will be accepted.
(*Answer:* 0·592 lead; 125 A; 2·97 MW)

6. A 3-phase, star-connected turbo-alternator with a synchronous reactance of 8 ohms per phase delivers 200 A at unity power-factor when connected to 11 kV constant frequency busbars. If the throttle is held constant while the excitation e.m.f. is increased by 30% determine the new current and power-factor. The excitation e.m.f. is now held constant at the new value and the steam supply is gradually increased. Find the value of output power at which the machine will break from synchronism and the operating power-factor under these conditions. Neglect armature resistance.
(*Answer:* 324A; 0·622 lag; 19·8 MW; 0·6 lead)

7. A 6·6 kV, 50 Hz, 3-phase, star-connected synchronous motor has an open-circuit characteristic given by

Field current p.u.	0·80	1·0	1·20	1·40	1·60
Open-circuit voltage p.u.	0·83	1·0	1·13	1·23	1·30

The unsaturated synchronous reactance is 200 Ω and resistance can be neglected.

Find the per-unit field current required for a b.h.p. of 100 at efficiency 0·88 when the operating power factor is 0·9 lead.

(*Answer:* 1·51)

8. An over-excited 3-phase, 50 Hz, star-connected synchronous motor with a synchronous reactance of 100 ohms per phase and negligible resistance is to be used to improve the overall power-factor of a system supplying a factory to unity, and to provide a constant speed drive with an additional 50 h.p. The existing factory load is 100 kVA at power-factor 0·6 lagging supplied from 6·6 kV, 50 c/s, 3-phase, infinite busbars. Determine the rating, operating power-factor and the value of excitation e.m.f. for the synchronous motor if, under these conditions, the per-unit efficiency is 0·8. Describe a suitable method of starting such a motor.

(*Answer:* 92·5 kVA; 0·5 lead; 4540 V)

9. Show that the total reactive volt.amperes Q for a cylindrical rotor machine are given by

$$Q = 3\frac{V^2}{X_s} - \frac{VE_f}{X_s}\cos\delta.$$

Determine the maximum value of Q the machine can deliver with fixed excitation. Resistance can be neglected.

10. A 3-phase, salient pole synchronous motor operates at rated voltage on infinite busbars. If the per-unit values of the synchronous reactances are $X_d = 1{\cdot}0$, $X_q = 0{\cdot}8$ and resistance can be neglected, calculate the percentage of rated power the machine can develop with (a) zero excitation, (b) 0·8 p.u. excitation. Find the corresponding load angle and armature current in each case.

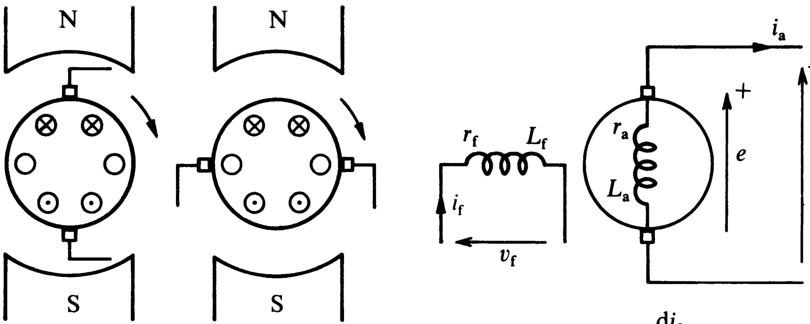
(*Answer:* 0·125; 45°; 1·26; 0·834; 74·65°; 1·26)

11. Construct the phasor diagram for a 20 MVA, 3-phase, star-connected, 50 Hz, salient pole synchronous generator with $X_d = 1$ p.u., $X_q = 0{\cdot}65$ p.u., $R_a = 0{\cdot}01$ p.u. delivering 15 MW at power-factor 0·8 lag to an 11 kV, 50 Hz system. Find the load angle and per-unit excitation e.m.f. under these conditions.

(*Answer:* 18°; 1·73)

8. The d.c. machine

The basic principles underlying the operation of d.c. machines have been discussed in chapter 1 where it has been noted that the field winding is mounted on salient poles on the stator and the armature winding is wound in slots on a cylindrical rotor. The frequency of the e.m.f. generated in the armature depends directly on the mechanical speed of rotation, and the action of the commutator is that of a frequency changer so that the frequency at the commutator brushes is zero in this particular case. Various arrangements of



$$v_f = i_f r_f + L_f \frac{di_f}{dt}$$
$$v = e - \left(i_a r_a + L_a \frac{di_a}{dt}\right)$$

(a) (b) (c)

***Fig. 8.1.** Physical and schematic arrangements: (a) Physical. (b) Schematic 1. (c) Schematic 2.*

armature windings have been discussed in chapter 2 where it has been noted that the commutator brush is normally situated on the centre line of a main pole, although it is connected to a coil in the interpolar gap. It is usual to use a schematic representation of a d.c. machine in which the brushes are shown in the position of the coil to which they are connected, and the physical arrangement together with the two schematic diagrams to be used in this text are shown in Fig. 8.1 for the particular case of a generator. The torque angle θ defined in chapter 1 will normally be 90° so that, under steady state conditions, any difference between generated e.m.f. and terminal voltage is governed by armature resistance voltage drop. The generated e.m.f. E is given from (3.35) as

$$E = \frac{pz}{2\pi a}\Phi\omega = K\Phi\omega \tag{8.1}$$

and the electromagnetic torque, T_e, from (3.51) as

$$T_e = \frac{pz}{2\pi a}\Phi I_a = K\Phi I_a. \tag{8.2}$$

In the case of a motor, the terminal voltage will always be numerically greater than the generated e.m.f. and the machine torque will produce rotation against a load. For a generator, the terminal voltage will be less than the generated e.m.f. and the machine torque will oppose that applied to the shaft by the prime mover.

8.1. Armature reaction

Any current in the armature conductors must produce an m.m.f. with an associated flux, and this flux, the so-called flux of armature reaction will combine with the field flux to produce the resultant flux of the machine. A developed diagram showing the component and total m.m.f.s and fluxes in a 2-pole machine is given in Fig. 8.2. The m.m.f. of the armature current will be of a stepped form shown in Fig. 8.2(b) which can be approximated to the triangular form shown. The resulting armature reaction flux is shown in Fig. 8.2(b) and is low in the interpolar gaps because of the high reluctance of this area. The form of the resultant flux is shown in Fig. 8.2(c) and its magnitude will be less than that produced by the field alone. It can be seen from Fig. 8.2(c) that the effect of armature reaction is to distort the flux wave and to shift the position of the magnetic neutral axis (m.n.a.) in the direction of rotation for the generating case and against the direction of rotation for the motoring case.

The effect of total flux reduction by armature reaction is known as the demagnetizing effect and this effect can be completely removed by the use of a distributed compensating winding arranged in slots in the main poles so that the effects of the m.m.f. set up by currents in armature conductors situated under the main poles is cancelled. Thus the compensating winding is connected in series with the armature winding and the total m.m.f. per pole of this winding must equal the m.m.f. of that portion of the armature under any one pole, i.e., be equal to the number of armature conductors per pole per parallel path times the ratio pole arc–pole pitch.

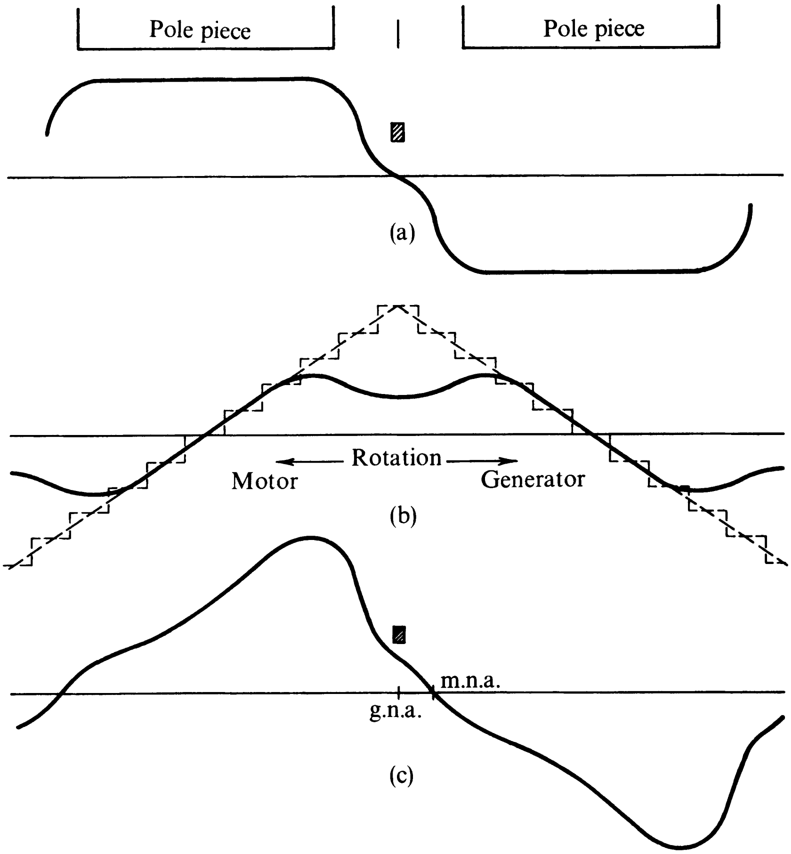


Fig. 8.2. Component and total m.m.f. and flux:
(a) Field alone. (b) Armature alone. (c) Resultant.

8.2. The commutation process

When the commutator bar which is connected to a particular armature coil rotates past a fixed commutator brush it follows that the coil current must reverse. In many cases, a brush will span more than one commutator segment, so that, during commutation, several coils in series will be short-circuited. The reversal of the coil current induces a voltage of self-inductance, the so-called reactance voltage, which opposes the change of current so that a spark could appear at the trailing edge of the brush. The ideal process of commutation is illustrated in Fig. 8.3 and the reactance voltage will produce the condition known as 'under commutation' shown in Fig. 8.4.

It can be seen from Fig. 8.2 that one of the reasons for poor commutation is the effect of armature reaction flux in shifting the position of the magnetic neutral axis. This flux distortion and the reactance voltage will always exist, but their effects on commutation can be countered by the introduction of a so-called 'commutation voltage'. In the case of small machines, this voltage can be introduced by shifting the brush position but the most general method is to insert small auxiliary poles centred on the interpolar gap, known as *compoles*, connected in series with the armature. Since the compole must

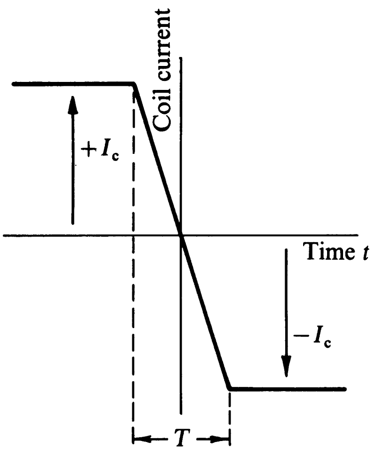


Fig. 8.3. Ideal commutation.

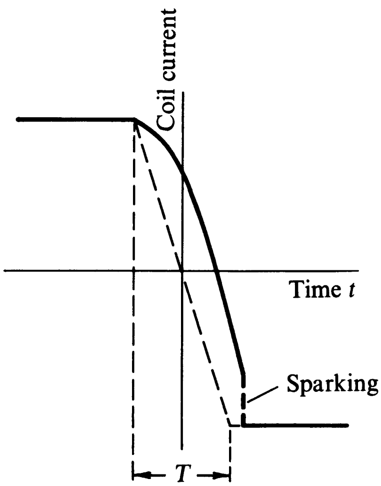


Fig. 8.4. Under commutation.

produce flux proportional to armature current, it must operate with an unsaturated magnetic circuit, and the air-gap between the armature and the compole will normally be greater than that between the armature and the main-pole. The arrangement of a 2-pole generator with compoles is shown in Fig. 8.5 and the compole has the same polarity as the main pole ahead in the direction of rotation. In the case of a motor, the arrangement is shown in Fig. 8.6 and the compole has the same polarity as the main pole behind in the direction of rotation. In these circumstances, with the correct design of

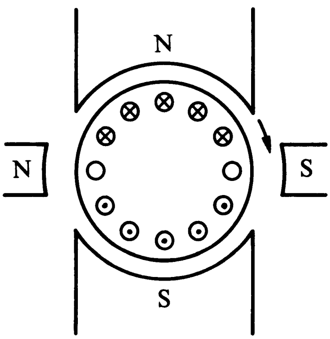


Fig. 8.5. Generator with compoles.

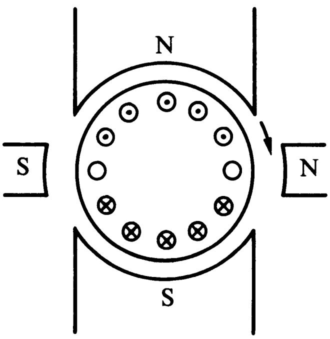


Fig. 8.6. Motor with compoles.

compole, a d.c. machine can be operated as either a motor or a generator with either direction of rotation.

A developed diagram showing the component m.m.f. and flux patterns for a generator fitted with a compensating winding and compoles is shown in Fig. 8.7, and it can be seen that the total flux is almost the same as that of the field winding acting alone.

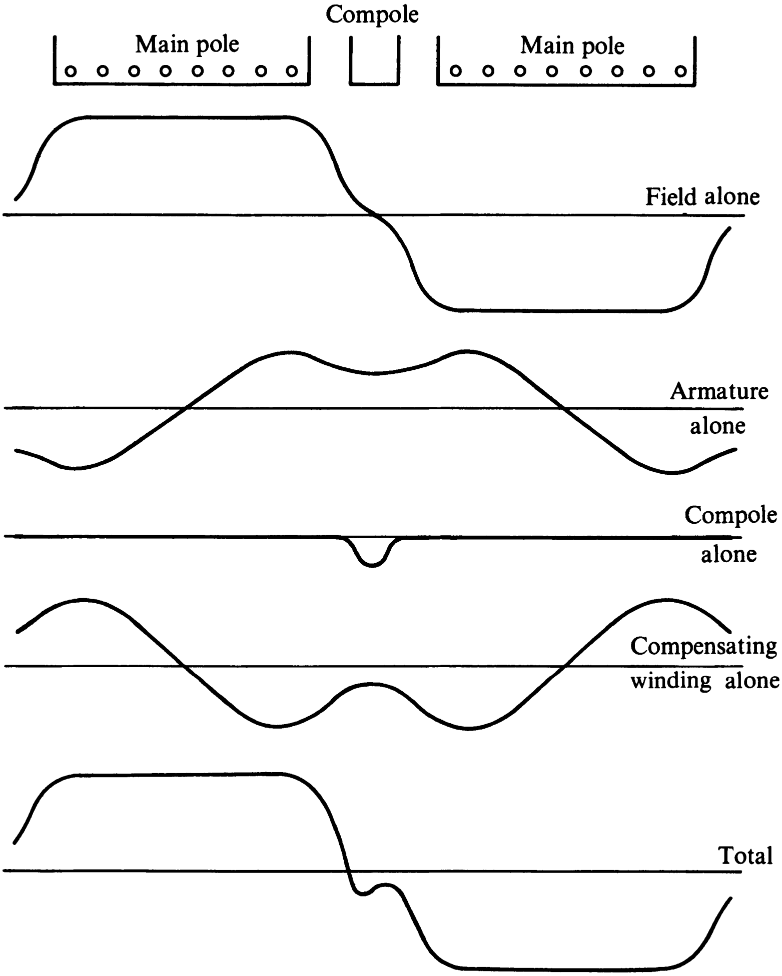


Fig. 8.7. Component and total m.m.f. and flux for machine with compoles and compensating winding.

8.3. Generator performance characteristics

The terminal voltage V of a d.c. generator is related to the armature current I_a and the generated e.m.f. E, by

$$V = E - I_a R_a \qquad (8.3)$$

where R_a is the total internal armature resistance. The value of the e.m.f. E, is given by (8.1) and is governed by the field flux Φ and the angular velocity ω of the rotor. A curve relating the e.m.f. to the field current at constant speed is known as the magnetization curve or open-circuit characteristic. In the case of a separately excited machine, the field voltage is independent of the armature voltage; for a shunt connected machine the field is in parallel with the armature and for a series connected machine, the field is in series with the armature. It follows immediately that, for a shunt or series connected generator, the armature voltage will only build up if residual flux is present.

In general, the three characteristics which specify the performance of a d.c. generator are:

(i) the open-circuit characteristic,

(ii) the external characteristic which gives the relationship between the terminal voltage and load current at constant speed,

(iii) the load characteristic which gives the relationship between the terminal voltage and the field current, with constant armature current and speed.

All other characteristics depend on the form of the open-circuit characteristic, the load and the method of connection.

8.4. The separately excited generator

The schematic connection diagram for a separately excited generator is shown in Fig. 8.8 and the form of the open-circuit and load characteristics are shown in Fig. 8.9. When the voltage drop in the armature circuit, for a

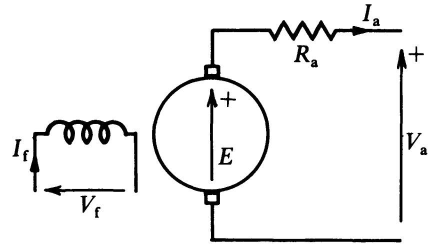


Fig. 8.8. Schematic diagram for separately excited generator.

given armature current, is added to the load characteristic the curve C shown in Fig. 8.9 is obtained. Then, at a given field current OA, the distance CD represents the voltage drop arising from armature reaction. The form of the external characteristic is shown also in Fig. 8.9, and the terminal voltage falls slightly as the load current increases.

Voltage regulation is defined as the percentage change in terminal voltage when full load is removed, so that, from Fig. 8.9

$$\text{Voltage regulation} = \frac{E - V}{V} \times 100\%. \tag{8.4}$$

Since the separately excited generator requires a separate d.c. field supply, its use is limited to applications where a wide range of controlled voltage is essential.

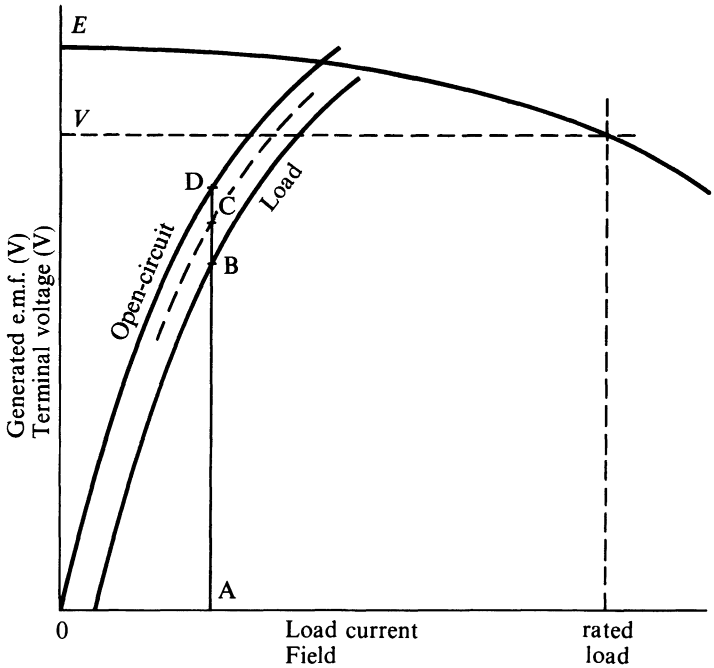


Fig. 8.9. Open-circuit and load characteristics.

8.5. The shunt generator

This machine operates with the field connected in parallel with the armature in the manner shown in Fig. 8.10. The open-circuit characteristic has the same form as that of the separately excited machine and, since this

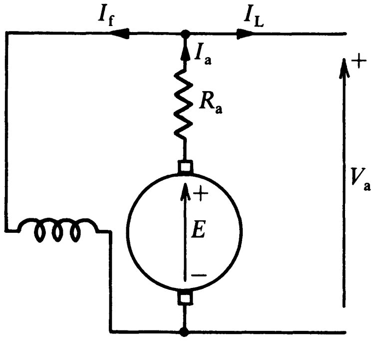


Fig. 8.10. Schematic diagram for shunt generator.

curve shows the relationship between the armature voltage V, and field current I_f, a field resistance line slope $R_f = V/I_f$ can be incorporated on the characteristic. Figure 8.11 shows the form of the characteristic with a field resistance R_f = OA/OB so that the open-circuit voltage will build up to a value OA. It follows that there must be a critical value of field resistance, such that the field resistance line is coincident with the linear part of the characteristic. This value of resistance, known as the critical field resistance, is shown

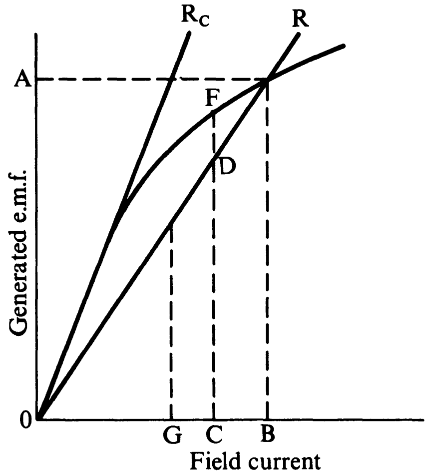


Fig. 8.11. Open-circuit characteristic.

as the line OR_C in Fig. 8.11 and is the maximum permissible value of field resistance if the armature voltage is to build up. For a value of field current given by OC in Fig. 8.11, the field resistance line OR can be drawn and the field voltage and, hence the terminal voltage, are given by CD. The generated e.m.f. is given by CF and hence the armature resistance drop is given by DF.

Various graphical techniques are available to determine the variation of terminal voltage with armature current when the form of the open-circuit characteristic, the field resistance and turns, the demagnetizing ampere turns and the armature resistance are known. One such technique is illustrated in Fig. 8.12. For an assumed value of armature current O′L, the effective reduction in total ampere turns produced by demagnetization is known. Then the equivalent reduction in field current produced by demagnetization can be set up as the length OA in Fig. 8.12, such that the length OA is the demagnetization ampere turns divided by the field turns. The armature resistance drop for the assumed value of armature current is given by the length AB in Fig. 8.12. A line through B parallel to the field resistance line OR is now drawn to meet the O.C.C. at the points C and D. Horizontal lines CM and DN are drawn to meet the vertical from L at points M and N. Then M and N are points on the terminal voltage–armature current characteristic and, if this process is repeated for several different values of armature current, the complete characteristic in the form shown in Fig. 8.12 can be obtained. The external characteristic can then be obtained by subtracting the field current I_f from the armature current for each point on the voltage–armature current characteristic. It can be seen from Fig. 8.12 that there is a maximum value of armature current so that, beyond this point, a further decrease in load resistance will produce a decrease in armature current. Under short-circuit conditions, the armature current will be the ratio of the residual voltage to the internal armature resistance. When the demagnetizing effect is neglected the distance OA in Fig. 8.12 becomes zero and the construction for the external characteristic is simplified.

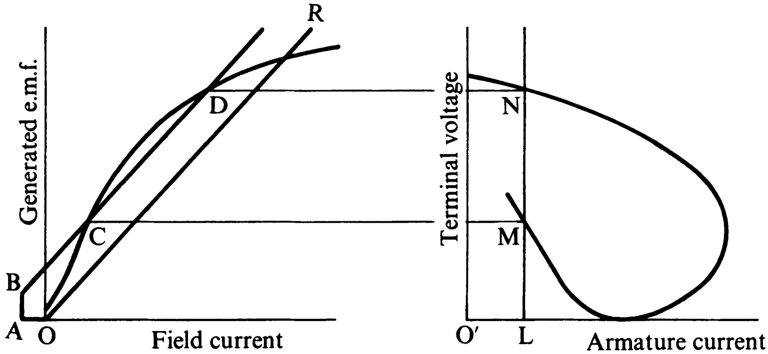


Fig. 8.12. Graphical technique to find V_a–I_a curve.

A shunt generator maintains approximately constant voltage on load and finds wide application as an exciter for the field circuit of a large generator and as a tachogenerator when a signal proportional to motor speed is required for control or display purposes.

8.6. Performance characteristics of motors

An electric motor must produce rotation of a shaft against a load torque T_L and the motor must, therefore, produce torque. The electromagnetic torque T_e of a d.c. motor is given by (8.2) as

$$T_e = \frac{pz}{2\pi a}\Phi I_a = K\Phi I_a \tag{8.5}$$

and it follows immediately from (8.5) that the torque of a d.c. motor can be controlled by variations in the armature current. In the case of a d.c. shunt motor, the field current can simply be controlled by the use of a variable resistance in series with the field winding. For a series motor, the armature and field circuits are connected in series and it is somewhat difficult to control the armature and field currents independently.

The terminal voltage V is given by

$$V = E + I_a R_a \tag{8.6}$$

where, by (8.1),

$$E = \frac{pz}{2\pi a}\Phi\omega.$$

It immediately follows that

$$EI_a = \omega T_e \tag{8.7}$$

where EI_a is the gross output power.

The forms of the torque–armature current, speed–torque, and speed–power characteristics for a shunt connected, series connected, and cumulatively compounded d.c. motor are illustrated in Fig. 8.13. Most motors

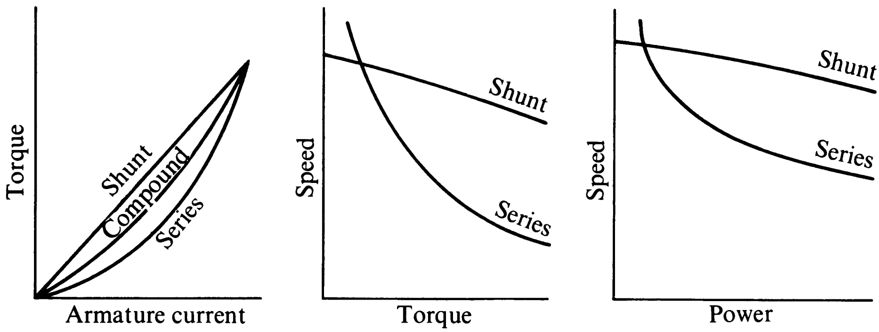


Fig. 8.13. Characteristic curves for motors.

will be designed to develop a given horse-power at a specified speed and it follows from (8.1) and (8.6) that the angular velocity ω can be written

$$\omega = \frac{V - I_a R_a}{K\Phi}. \tag{8.8}$$

Thus the speed of a d.c. motor depends on the values of the applied voltage V, the armature current I_a and resistance R_a, and the field flux per pole Φ.

8.7. The shunt motor

The arrangement of a shunt motor is given in schematic form in Fig. 8.14 and, if the applied voltage is constant, the field current and hence the flux Φ will be constant. Then, from (8.2) the torque T_e is given by

$$T_e = (K\Phi)I_a = K_m I_a. \tag{8.9}$$

In a similar manner, the generated e.m.f. is given by

$$E = (K\Phi)\omega = K_m\omega \tag{8.10}$$

so that, from (8.6), (8.9), and (8.10)

$$T_e = \frac{VK_m}{R_a} - \frac{K_m^2}{R_a}\omega \tag{8.11}$$

and, from (8.7) and (8.11)

$$EI_a = \omega T_e = \frac{VK_m\omega}{R_a} - \frac{K_m^2\omega^2}{R_a}. \tag{8.12}$$

The form of (8.11) and (8.12) confirm the form of the characteristic curves for the shunt motor shown in Fig. 8.13.

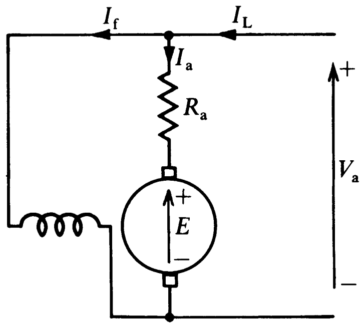


Fig. 8.14. Schematic diagram for shunt motor.

The shunt motor is essentially a constant speed machine with a low speed regulation. It can be seen from (8.8) that the speed is inversely proportional to the field flux, so that its speed can be varied by control of field flux. When, however, the motor operates at very low values of field flux, the speed will be high and, if the field becomes open-circuited the speed will rise rapidly beyond the permissible limit governed by the mechanical structure. If a shunt motor is designed to operate with a low value of shunt field flux, it will usually be fitted with a small cumulative series winding, known as a stabilizing winding, to limit the speed to a safe value.

8.8. The series motor

A schematic representation of the series motor is given in Fig. 8.15. The field flux is determined directly by the armature current and, if there is no saturation, flux is proportional to armature current so that, from (8.2)

$$T_e = K\Phi I_a = KI_a^2. \tag{8.13}$$

From (8.6), with negligible armature resistance,

$$V = E = K\Phi\omega = KI_a\omega \tag{8.14}$$

and it follows directly from (8.13) and (8.14) that

$$T_e = KI_a^2 = \frac{V^2}{K\omega^2}. \tag{8.15}$$

Thus

$$P_g = \omega T_e = \frac{V^2}{K\omega} \tag{8.16}$$

and the speed–power curve is a rectangular hyperbola. The forms of (8.13), (8.15), and (8.16) confirm the shape of the characteristic curves of the series motor shown in Fig. 8.13.

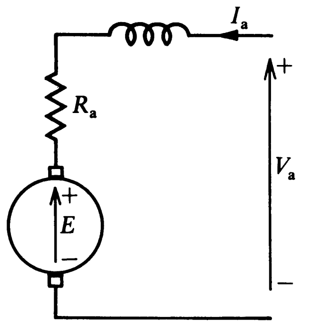


Fig. 8.15. Schematic diagram for series motor.

Figure 8.13 shows that the no-load speed is very high and care must be taken to ensure that the machine always operates on load. In practice, the series machine will normally have a small shunt field winding to limit the no-load speed. The assumption that flux is proportional to armature current is only valid on light load and, in general, performance characteristics of the series motor must be obtained using the magnetization curve.

This machine is ideally suited to traction where large torques are required at low speeds and relatively low torques are required at high speeds.

EXAMPLE 8.1. A 200 V d.c. shunt motor has a field resistance of 160 Ω and an armature resistance of 0·5 Ω. The motor operates on no-load with full field flux at its base speed of 800 r.p.m. with an armature current of 4 A. If the machine drives a load requiring a torque of 98 N.m, calculate the armature current and speed of the machine.

The machine is required to develop 7·5 h.p. at 1000 r.p.m. What is the required value of external series resistance in the field circuit?

Magnetic saturation and armature reaction can be neglected.

Solution: Full field current $I_f = 200/160 = 1{\cdot}25$ A.
On no-load, $E = V_a - I_aR_a = 200 - 4 \times 0{\cdot}5 = 198$ V. Also $E = KI_f\omega$.

$$\therefore\ K = \frac{198}{1{\cdot}25\left(\frac{2\pi}{60}800\right)} = 1{\cdot}895.$$

On load, $T_e = KI_fI_a$, that is, $98 = 1{\cdot}895 \times 1{\cdot}25 \times I_a$.
Armature current $I_a = 41{\cdot}1$ A.
Now $V = E + I_aR_a$, i.e., $E = 200 - 41{\cdot}4 \times 0{\cdot}5 = 179{\cdot}3$ V. But $E = KI_f\omega$,

$$\therefore\ \omega = \frac{179{\cdot}3}{1{\cdot}895 \times 1{\cdot}25} = 75{\cdot}7 \text{ rad/s}.$$

Load speed = 724 r.p.m.
For 7·5 h.p. at 1000 r.p.m.,

$$T_e = \frac{7{\cdot}5 \times 746}{\frac{2\pi}{60}1000} = 53{\cdot}2 \text{ N.m}.$$

Then $$53{\cdot}2 = 1{\cdot}895 I_f I_a, \quad \text{or} \quad I_fI_a = 28{\cdot}1.$$

Now $$V = E + I_aR_a,$$

i.e., $$200 = 1{\cdot}895\left(\frac{2\pi}{60}1000\right)I_f + 0{\cdot}5I_a$$

$$= 198{\cdot}5I_f + 0{\cdot}5I_a = 198{\cdot}5I_f + 0{\cdot}5\frac{28{\cdot}1}{I_f}.$$

Therefore, $I_f = 0{\cdot}935$ A or $0{\cdot}075$ A. Thus $I_a = 28{\cdot}1/I_f = 30$ A or 375 A.

The value of $I_f = 0{\cdot}075$ A will produce very high armature currents and will not be considered.

When $I_f = 0{\cdot}935$, $$R_f = \frac{200}{0{\cdot}935} = 214\,\Omega.$$

External resistance required $= 214 - 160 = 54\,\Omega$.

EXAMPLE 8.2. A 200 V d.c. series motor has the following O.C.C. measured at 1000 r.p.m.

Field current (A)	5	10	15	20	25	30
Open-circuit voltage (V)	80	160	202	222	236	244

The armature resistance is 0·25 Ω and the series field resistance is 0·25 Ω. Find the speed of the machine when (a) the armature current is 22·5 A, (b) the gross torque is 36 N.m.

Solution: The O.C.C. is plotted in Fig. 8.16.
In general, $E = K\Phi\omega$, so that

$$\frac{E_1}{E_2} = \frac{\Phi_1\omega_1}{\Phi_2\omega_2} = \frac{\Phi_1N_1}{\Phi_2N_2}.$$

(a) When $I_a = 22{\cdot}5$ A, $E_1 = 230$ V at $N_1 = 1000$ r.p.m. from Fig. 8.16. Now $E = V - I_aR = 200 - 22{\cdot}5 \times 0{\cdot}5 = 188{\cdot}75$ V. Then

$$\frac{230}{188{\cdot}75} = \frac{1000}{N} \quad \text{and} \quad \text{Speed } N = \frac{188{\cdot}75}{230} \times 1000 = 819 \text{ r.p.m.}$$

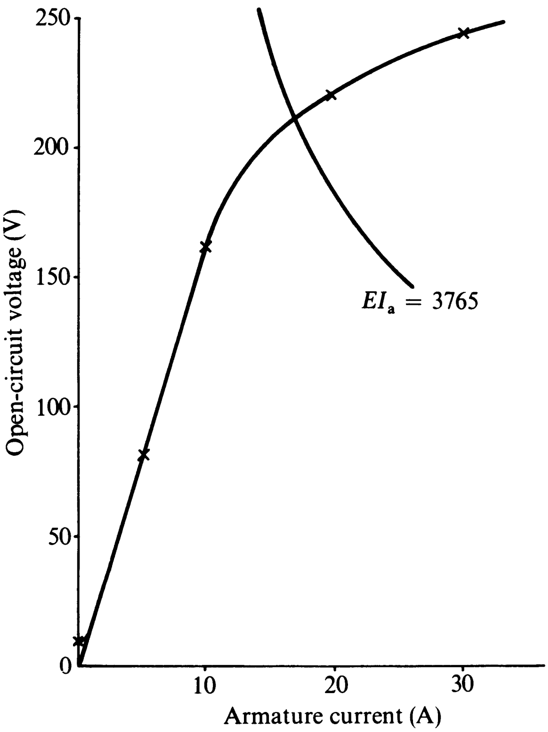


Fig. 8.16. Open-circuit characteristic for Example 8.2.

(b) When $T_e = 36$ N.m, $T_e = EI_a/\omega$. Then, at 1000 r.p.m.,

$$EI_a = 36\left(\frac{2\pi}{60} \times 1000\right) = 3765.$$

The rectangular hyperbola $EI_a = 3765$ is plotted on the O.C.C. in the manner shown in Fig. 8.16. At the point of intersection,

$I_a = 17{\cdot}4$, $E_1 = 212{\cdot}5$ V at 1000 r.p.m., $E = 200 - 17{\cdot}4 \times 0{\cdot}5 = 191{\cdot}3$ V.

Then $$\text{Speed } N = \frac{191{\cdot}3}{212{\cdot}5} \times 1000 = 902 \text{ r.p.m.}$$

8.9. The compound motor

A schematic diagram for a cumulatively compounded d.c. motor is shown in Fig. 8.17. The operating characteristics of this machine lie between those of the shunt and series motor. A series motor with a shunt limiting winding is, of course, a compound motor with essentially series characteristics, while a shunt motor with a series stabilizing winding is a compound motor with shunt characteristics.

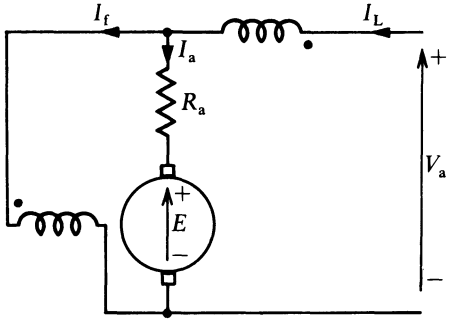


Fig. 8.17. Schematic diagram for compound motor.

The differentially compounded motor operates with the flux set up by current in the series winding opposing that set up by current in the shunt winding. On light load, the effect of the series winding will be small. On high loads, however, its effect will be to continuously reduce the total flux as the load increases. This has the effect of increasing the speed with a consequent increase in armature current, decrease in flux, and further increase in speed. Such an arrangement will therefore be unstable and, in these circumstances, the differentially compounded motor has little application.

8.10. Speed control of motors

It has been shown in section 8.6 that the speed of a motor can be varied by control of the armature applied voltage, the armature resistance, or the field flux. The base speed of the machine is defined as the speed with rated armature voltage and normal armature resistance and field flux.

Speed control above the base value can be obtained by variation of field flux. In the case of a machine with a high resistance shunt field winding, speed control over a wide range above the base speed can be obtained by inserting a series resistance in the field circuit. It is important to note that a reduction in field flux produces a corresponding increase in speed, so that the generated e.m.f. does not change appreciably as the speed is increased. However, the machine torque is reduced as the field flux is reduced, so that this method of speed control is suited to applications where the load torque falls as the speed increases. In the case of a machine with a series field, speed control above the base value can be obtained by placing a diverting resistance in parallel with the series winding so that the field current is less than the armature current.

When speed control below the base value of speed is required, the effective armature resistance can be increased by inserting external resistance in series with the armature, and this method can be applied to shunt series or compound motors. It has the disadvantage however, that the resistances required must take full armature current and there will, therefore, be a large external power loss with an associated reduction in overall efficiency. The speed of the machine will be governed by the value of the voltage drop in the series resistor and will therefore be a function of the load on the machine. In these circumstances, the application of this method of control is limited although it has some use where reduced speed is required intermittently.

8.11. The Ward-Leonard method of speed control

This system is one of the most versatile methods of speed control available and involves the use of a separate motor-generator set to supply variable voltage to the armature of the machine under control. The arrangement is illustrated in Fig. 8.18 in which machine A is usually a 3-phase, squirrel-cage

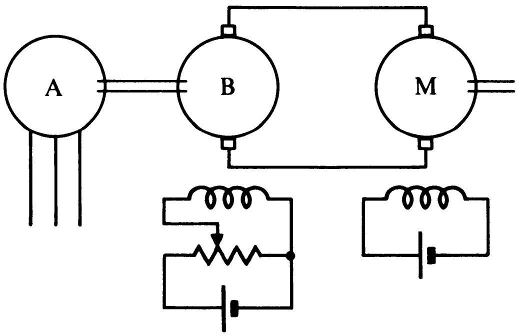


Fig. 8.18. Ward-Leonard system.

induction motor driving the separately excited d.c. generator B whose armature is electrically connected to that of the separately excited d.c. motor M under control. The field of machine B is supplied with a variable voltage and that of the main motor M with a fixed voltage. These field voltages can be obtained from a small excitor in the form of a shunt generator driven by machine A. As the excitation of machine B is varied, so its generated e.m.f. and, hence, the speed of the main motor varies. Thus, continuous speed control is available over very wide ranges with the major advantages that large torques are available at low speeds. This arrangement also has the advantage that, when the armature voltage of the main motor is reduced to produce a decrease in speed, the motor M will momentarily operate as a generator and machine B will operate as a motor driving machine A as a generator. In this manner, the change in kinetic energy of the main motor during retardation can be converted to electrical energy and returned to the power system. Such a process is known as regeneration.

8.12. Motor starters

When voltage is applied to the armature of a d.c. motor with the machine stationary, there will be no generated e.m.f. and the armature current will only be limited by the internal armature resistance of the machine. It is, therefore, usual to incorporate a variable resistance in series with the armature which is gradually decreased to zero as the motor accelerates. Typical forms of manual starters for d.c. motors are shown in Fig. 8.19. The three-

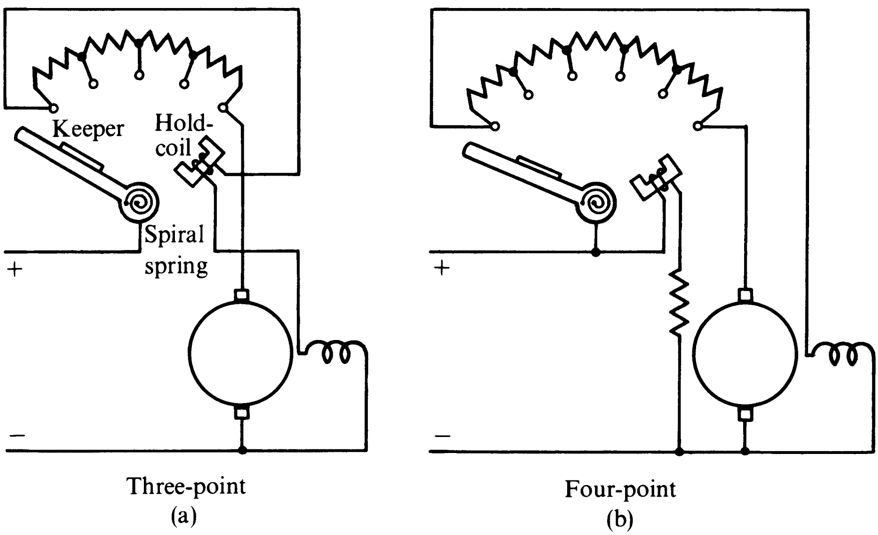


Fig. 8.19. Typical forms of manual starter: (a) Three point. (b) Four point.

point starter shown in Fig. 8.19(a) has the hold-coil connected in series with the field circuit so that the starter arm, which is spring loaded, drops back to the off position if the field circuit is open-circuited. In the case of the four-point starter shown in Fig. 8.19(b), the hold-coil is connected directly across the supply so that the starter arm drops back to the off position if the voltage supply to the machine is removed. In both cases, the field circuit is completed when the moving arm is connected to the first stud of the variable resistor and it is usual to include a relay in the armature circuit to give overload protection.

If facilities for changing the field current are incorporated within the starter, it is known as a controller. It is usual to incorporate mechanical interlocks in the controller to ensure that the machine can only be started with full field flux. The drum controller for a series motor will provide facilities for starting, speed control, and reversal of the direction of rotation and it follows that the external series resistor must be rated to carry full-load current continuously.

8.13. Losses and efficiency

The efficiency of a d.c. machine can be written as

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}. \quad (8.17)$$

The losses are made up of copper loss in the armature and field circuits, brush contact resistance, iron losses, friction and windage losses, and stray load losses. It is usual to determine the efficiency by some method based on the measurement of losses.

One such method is known as Swinburne's method in which the machine is run on no-load at rated speed with rated applied voltage. The armature resistance R_a is measured and the armature copper loss is calculated for an assumed armature current I_a. The remainder of the losses are assumed to be constant and independent of the armature current. For a no-load applied voltage V, armature current I_o, and field current I_f, the efficiency as a motor is given by

$$\text{Efficiency} = 1 - \frac{V(I_f + I_o) + I_a^2 R_a}{V(I_f + I_a)}. \quad (8.18)$$

The corresponding efficiency as a generator is given by

$$\text{Efficiency} = 1 - \frac{V(I_f + I_o) + I_a^2 R_a}{V(I_a + I_o) + I_a^2 R_a}. \quad (8.19)$$

The value of efficiency obtained by this method is usually greater than the actual efficiency.

A more accurate method of predicting the efficiency is the Kapp-Hopkinson Test for which two similar machines mechanically coupled and electrically connected back-to-back are required. The method of connection is illustrated in Fig. 8.20. Machine A is run up to normal speed as a motor with machine B unexcited. Excitation is then applied to machine B and increased until the open-circuit voltage of this machine is equal to the system voltage. Machine B is then connected to the system and, if the excitation of machine B is increased when that of machine A is decreased so that the speed is unchanged, machine A will motor while machine B generates. In these circumstances the supply system will provide the total losses.

Then, from Fig. 8.20,

$$\text{Generator input} = \text{Motor output} = \eta \times \text{Motor input} = \eta V I_A$$

where η is the efficiency of each machine.

Generator output $= VI_B = \eta \times$ Generator input, so that $\eta^2 VI_A = VI_B$. Then,

$$\text{Efficiency } \eta = \sqrt{\frac{I_B}{I_A}}. \tag{8.20}$$

It can be seen from Fig. 8.20 that the quantity $P = VI_L - (I_A{}^2R_A + I_B{}^2R_B)$ represents the total fixed loss of the system so that the fixed loss of

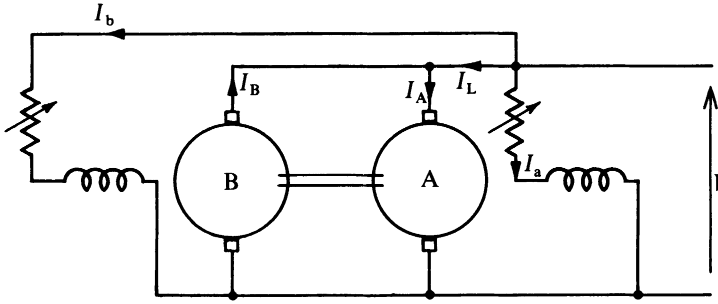


Fig. 8.20. Kapp-Hopkinson test.

each machine can be assumed to be $\frac{1}{2}P$. The input power to the motor is $VI_A + VI_a$ and the output power from the generator is VI_B. Then

$$\text{Motor efficiency} = 1 - \frac{(P/2) + I_A{}^2R_A + VI_a}{V(I_A + I_a)} \tag{8.21}$$

and
$$\text{Generator efficiency} = 1 - \frac{(P/2) + I_B{}^2R_B + VI_b}{V(I_B + I_b) + (P/2) + I_B{}^2R_B}. \tag{8.22}$$

The values of efficiency found from (8.21) and (8.22) will be close to the actual efficiency.

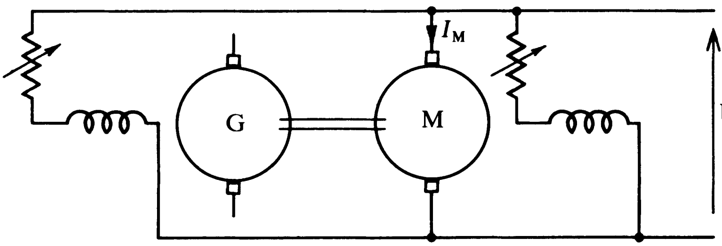


Fig. 8.21. Separation of fixed losses.

When it is necessary to obtain measured values for the separate iron losses and friction and windage losses for a machine, the arrangement shown in Fig. 8.21 can be used. The machine under test, G, is driven by an auxiliary d.c. motor M, the excitation of M is set to produce the rated speed for G, and the excitation of G is set to produce rated voltage on open-circuit. The speed of the set is then increased to approximately 125% of rated speed by increasing the field resistance of the motor and the two excitations are then maintained constant. Increasing armature resistance is used to decrease the speed of the set in steps and the quantity $W_1 = VI_M - I_M{}^2R_M$ (where R_M is the armature resistance of the motor) is measured at each step. This process is repeated with the generator unexcited to obtain W_2, and with the generator uncoupled to obtain W_3. The iron losses of the generator are then equal to $W_1 - W_2$ and the friction and windage losses of the generator are $W_2 - W_3$.

8.14. Dynamics of separately excited motors

The transient performance of d.c. generators and motors is of some importance when considering the operation of closed loop control systems, and many position control servomechanisms utilize a separately excited d.c. motor as the main control element.

A schematic diagram representing this machine is shown in Fig. 8.22. The machine will normally operate with constant field current I_f so that the generated e.m.f. e can be written

$$e = KI_f\omega = K_m\omega. \tag{8.23}$$

The torque t_e can be written

$$t_e = KI_fi_a = K_mi_a \tag{8.24}$$

where K_m is known as the motor torque constant and has the dimensions of

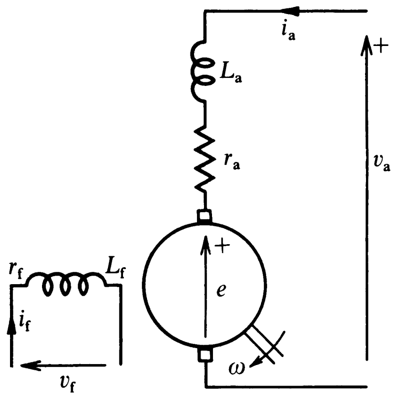


Fig. 8.22. The separately excited d.c. motor.

N.m per ampere. The voltage–current relationship for the armature can be written

$$v_a = e + (r_a + L_ap)i_a. \tag{8.25}$$

In many cases, the armature inductance can be neglected.

Consider the operation of the machine when a step voltage $V\,\mathrm{H}(t)$ is applied to the armature with constant field current. Then, from (8.23) and (8.25)

$$V\,\mathrm{H}(t) = K_m\omega + r_a i_a. \tag{8.26}$$

Now from (8.24)

$$t_e = K_m i_a = Jp\omega + T_L \tag{8.27}$$

so that, from (8.26) and (8.27) with the motor on no-load,

$$K_m\left[\frac{V\,\mathrm{H}(t) - K_m\omega}{r_a}\right] = Jp\omega, \quad \text{or} \quad \frac{K_m}{r_a}\left[\frac{V}{s} - K_m\bar{\omega}\right] = Js\bar{\omega}.$$

Then

$$\bar{\omega} = \frac{V}{s\left(K_m + \dfrac{r_a Js}{K_m}\right)}.$$

On solution,

$$\omega = \frac{V}{K_m}(1 - \mathrm{e}^{-t/\tau_m}) \tag{8.28}$$

where $\tau_m = Jr_a/K_m^2$ is the motor time constant. It follows from (8.28) that the speed rises exponentially when a step voltage is applied to the armature with the motor on no load.

When the load torque T_L can be written in the form $T_L = a + b\omega$, where a and b are constants, the above method of solution can be directly applied to obtain the speed as a function of time. When, however, the load torque T_L is given by $T_L = a + b\omega + c\omega^2 + \cdots + n\omega^2$, where $a, b, c, \ldots$ are constants and n is an integer, the resulting equations are non-linear differential equations. The solution of such equations is beyond the scope of this text, although graphical techniques leading to the solution of this type of problem are available.

When the separately excited d.c. motor is rheostatically braked, the arrangement can be represented by the schematic diagram shown in Fig. 8.23. With the motor initially on no-load, the operating equations can be written

$$e = K_m\omega = -i_a(r_a + r_L) \tag{8.29}$$

where r_L is the external braking resistance.

The machine torque t_e is given by (8.27) as $t_e = K_m i_a = Jp\omega$. Then, from (8.27) and (8.29)

$$K_m\left[\frac{-K_m\omega}{r_a + r_L}\right] = Jp\omega. \tag{8.30}$$

If the machine is initially operating at a speed ω_0, it follows from (8.30) that

$$\frac{-K_m^2\bar{\omega}}{r} = J(s\bar{\omega} - \omega_0), \quad \text{where} \quad r = r_a + r_L.$$

Then $$\bar{\omega}\left(Js + \frac{K_m^2}{r}\right) = J\omega_0, \quad \text{or} \quad \omega = \omega_0\, e^{-tK_m^2/Jr}. \tag{8.31}$$

Thus, from (8.31), the speed of the machine falls exponentially.

When the load is a friction load in the form $T_L = b\omega$, (8.27) can be written

$$t_e = K_m i_a = Jp\omega + b\omega. \tag{8.32}$$

Then from (8.29) and (8.32), $-K_m^2\omega/r = Jp\omega + b\omega$ so that $-K_m^2\bar{\omega}/r = J(s\bar{\omega} - \omega_0) + b\bar{\omega}$.

On solution, $$\omega = \omega_0 \exp -\left(b + \frac{K_m^2}{r}\right)\frac{t}{J}. \tag{8.33}$$

Comparison of (8.31) and (8.33) shows that the effect of the friction load is to decrease the time taken for the motor to stop.

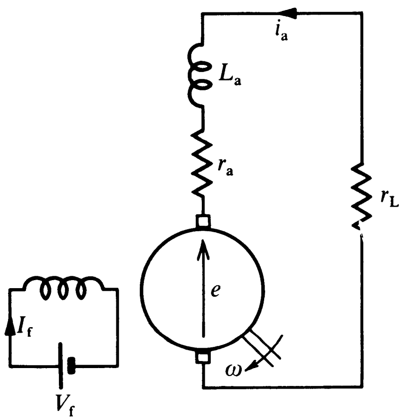


Fig. 8.23. Rheostatic braking.

The process of plugging a motor involves reversing the polarity of the supply to the armature of the machine. With the machine on no-load, the operating equations can be written

$$v_a = \frac{Jr_a}{K_m}p\omega + K_m\omega. \tag{8.34}$$

The initial speed ω_0 is given by $\omega_0 = V/K_m$, where V is the armature voltage.

When the machine is plugged, the armature voltage is reduced to zero and then reversed. This corresponds to a step voltage of $-V\,\mathrm{H(t)}$ with an initial speed of $\omega_0 = V/K_m$. Then, from (8.34),

$$\frac{-V}{s} = \frac{Jr_a}{K_m}(s\bar{\omega} - \omega_0) + K_m\bar{\omega}$$

so that $$\frac{Jr_a}{K_m}\omega_0 - \frac{V}{s} = \bar{\omega}\left(\frac{sJr_a}{K_m} + K_m\right).$$

On solution, with $\omega_0 = V/K_m$

$$\omega = \frac{V}{K_m}(2\,e^{-t/\tau_m} - 1). \tag{8.35}$$

When $t = 0$, $\omega = V/K_m = \omega_0$; when $t = \infty$, $\omega = -V/K_m = -\omega_0$.

An alternative method of analysis under plugging conditions is to use the method of superimposed change. Initially, with the machine on no-load $v_a = V$, $i_a = 0$, $\omega = \omega_0 = V/K_m$.

Finally, $v_a = -V$, $i_a = 0$, $\omega = \omega_F$, so that, for the change, $v_a = -2V\mathrm{H(t)}$, $\omega = \omega_t$. Then

$$v_a = -2V\,\mathrm{H(t)} = K_m\omega_t + i_a r_a, \quad \text{or} \quad -2V\,\mathrm{H(t)} = K_m\omega_t + (Jp\omega_t)\frac{r_a}{K_m}.$$

On solution, $$\omega_t = \frac{-2V}{K_m}(1 - e^{-t/\tau_m}).$$

When $t = \infty$, $\omega_t = 2V/K_m = -2\omega_0$.

Final velocity $\omega_F = \omega_0 + \omega_t = -V/K_m = -\omega_0$.

EXAMPLE 8.3. A separately excited 200 V d.c. motor operates at rated voltage with constant excitation on no-load. The torque constant of the motor is 2 N.m/A, its armature resistance is 0·5 Ω and the total moment of inertia of the rotating parts is 5 kg.m^2.

Derive an expression for the speed of the machine after plugging and find the time taken for the machine to stop. Fixed losses can be neglected.

Solution: On no-load, $E = V_a = 200 = K_m\omega_0$. No-load speed $\omega_0 =$ 100 rad/s.

Plugging corresponds to applying a step voltage -200 H(t) to the armature of the machine with, as initial speed, $\omega_0 = 100$. Then

$$v_a = e + i_a r_a = K_m\omega + i_a r_a, \qquad T_e = K_m i_a = Jp\omega$$

so that $$v_a = K_m\omega + \frac{r_a J}{K_m}p\omega$$

$$\frac{-200}{s} = K_m\bar{\omega} + \frac{r_a J}{K_m}(s\bar{\omega} - \omega_0)$$

$$= \left(2 + \frac{0{\cdot}5 \times 5}{2}s\right)\bar{\omega} - \frac{0{\cdot}5 \times 5}{2} \times 100$$

or $$\bar{\omega} = \frac{100}{s + 1{\cdot}6} - \frac{100}{s} + \frac{100}{s + 1{\cdot}6}.$$

Motor speed $\omega = 200\,e^{-1{\cdot}6t} - 100$.

Check: $t = 0, \omega = \omega_0 = 100; t = \infty, \omega = -\omega_0 = -100.$
When $\omega = 0; 0 = 200\,e^{-1\cdot 6t} - 100$, so that $1\cdot 6t = \log_e 2$.
Time for motor to stop = 0·347 sec.

Tutorial Problems

1. The following data were obtained for the magnetization curve of a 4-pole, shunt generator with compoles when run at 1500 r.p.m.

Field current (A)	0	0·1	0·4	0·6	0·8	1·0	1·14	1·32	1·56	1·92	2·4	3·04
Open-circuit voltage (V)	6	20	80	120	160	200	220	240	260	280	300	320

Draw the magnetization curve and hence obtain (a) the voltage to which the machine will build up as a self-excited generator, if the total shunt-field resistance is 125 ohms; (b) the critical value of shunt field resistance; (c) the magnetization curve for a speed of 1000 r.p.m.; (d) the open-circuit voltage of the generator when run at 1000 r.p.m. with a shunt field resistance of 125 ohms; (e) the speed for which a field resistance of 125 ohms is critical.

(*Answer:* 298 V; 200 ohms; 155 V; 938 r.p.m.)

2. If the armature resistance of the machine, whose magnetization curve at 1500 r.p.m. as given in Problem 1, is 0·4 ohm, calculate the terminal voltage of the machine when the armature current is 40 A and the field resistance is 125 ohms. Neglect armature reaction.

If armature reaction is equivalent to a reduction in field current of $0{\cdot}0025 I_a$ where I_a is the armature current, plot the terminal voltage–armature current relationship.

3. A long shunt cumulative compound d.c. generator has a shunt field of resistance 75 ohms with 700 turns per pole. The series winding of resistance 0·1 ohm has 6 turns per pole and the armature resistance is 0·25 ohm. Armature reaction is equivalent to a reduction of $0{\cdot}002 I_a$. The O.C.C. is given by

$$V_{oc} = 190 + 32(AT)_f$$

where V_{oc} is the generated e.m.f. and $(AT)_f$ is the field ampere turns for $4000 < (AT)_f < 7000$.

Plot the terminal voltage-armature current relationship.

4. A d.c. generator has an open-circuit characteristic given by $V_{oc} = 200 I_f/(K + I_f)$ such that the open-circuit voltage V_{oc} is 150 V when the field current I_f is 1·5 A. Find the critical field resistance and the open-circuit voltage when the field resistance is 200 ohms.

(*Answer:* 400 ohms; 100 V)

5. If the armature resistance of the machine described in Problem 4 is 0·5 ohm, calculate the terminal voltage and load current when the armature is connected to a load of resistance 10 ohms with a field resistance of 100 ohms.

(*Answer:* 138 V; 13·8 A)

6. A 200 V, 10 kW, 4-pole, long-shunt compound d.c. generator has armature, compole, series field, and shunt field resistances of 0·1, 0·03, 0·07, and 100 ohms respectively. Calculate the value of the generated e.m.f. when the machine is operating at full load. If the full-load speed is 1500 r.p.m. and the armature is lap-wound with 720 conductors, determine the flux per pole under the above conditions.

(*Answer:* 210·4 V; 11·67 mWb)

7. Two shunt generators operate in parallel to supply a total load current of 3000 A. Each machine has an armature resistance of 0·05 ohm and a field resistance of 100 ohms. If the generated e.m.f.s are 200 V and 210 V respectively, find the load voltage and the armature current of each machine.

(*Answer:* 1401 A; 1601 A; 130 V)

8. The external characteristics of two shunt generators in parallel are as follows:

Load current (A)		0	5	10	15	20	25	30
Terminal	I	270	263	254	240	222		
voltage (V)	II	280	277	270	263	253	243	228

Find the load current and terminal voltage of each machine when
(i) the generators supply a load of resistance 6 ohms
(ii) the generators supply a battery of e.m.f. 200 V and resistance 1·5 ohms.

(*Answer:* 240 V, 15 A, 26 A; 248 V, 12·2 A, 22 A)

9. A d.c. shunt machine with an armature resistance of 0·1 ohm and a field resistance of 100 ohms operates from a 200 V supply. If the machine operates with a line current of 60 A, find the ratio of the speed as a generator to that as a motor. Saturation can be neglected.

(*Answer:* 1:1·06)

10. Two similar shunt machines are tested by the Hopkinson method. When connected to a 240 V d.c. supply, and the generator is delivering an output of 42 kW, the current taken from the supply is 40 A. The shunt-field circuit resistance values of the motor and generator respectively are 300 and 240 ohms; the armature circuit resistance of each machine is 0·03 ohm. Calculate the efficiency of the generator, stating any assumption made.

(*Answer:* 0·916 approximate; 0·894 exact)

11. A 250 V shunt motor with an armature resistance of 0·25 ohm and a shunt field resistance of 250 ohms takes 4 A when running light. Calculate the efficiency when the armature current is 60 A.

(*Answer:* 0·873)

12. A 200 V, 10 h.p., d.c. shunt motor has an armature resistance of 0·35 ohms and a shunt field resistance of 200 ohms. The machine runs at 1750 r.p.m. with an efficiency of 87% on full-load. Calculate the no-load armature current and speed.

If a 2 ohm resistor is placed in the armature circuit to lower the speed, calculate the speed when the motor is developing full-load torque.

(*Answer:* 1885 r.p.m.; 964 r.p.m.)

13. A d.c. machine is separately excited from a variable voltage source and its armature is supplied through a resistor R ohms from a separate constant voltage source E_0 volts. The load on the machine has a torque requirement T N.m proportional to the speed N r.p.m. raised to the power p. Derive an expression relating the speed of the machine to the voltage across its field, and show that the speed will be a maximum when the armature current I_a is equal to $E_0/2R$. Saturation can be neglected.

14. A series motor has a total resistance of 0·25 ohm. When the motor is connected across a 230 V, d.c. supply it runs at 700 r.p.m. and develops a gross torque of 150 N.m.

Calculate the speed at which it will run and the gross torque it will develop when it is connected to a 240 V d.c. supply and the armature current is 35 A.

(*Answer:* 1045 r.p.m.; 76·2 N.m)

15. A 200 V series motor has an O.C.C. given by $V_{oc} = 1{\cdot}2N[I/I + 20)]$ where N is the speed in r.p.m. and I is the armature current. The internal resistance is 0·5 ohm and the armature current on full load is 30 A. Calculate the torque and speed at full load and the speed and armature current at half full load.

(*Answer:* 255 r.p.m.; 208 N.m.; 243 r.p.m.; 11·5 A)

16. A d.c. series motor has a rated output of 500 b.h.p. at 750 r.p.m. At full load the input current is 660 A at 600 V. Calculate the torque developed by the motor when the speed increases to cause the input current to rise to 725 A assuming that the resultant flux rises 40% as much as the current. If the resistance of the armature and compoles of this machine is 0·5 ohm, calculate the value of the external resistance to be added to the armature circuit if full-load torque is to be developed at standstill. Assume that flux is proportional to current.

(*Answer:* 2290 N.m; 0·409 ohm)

17. A 220 V, d.c. series motor takes 60 A when developing its rated output at 1000 r.p.m. The armature resistance is 0·12 ohm and the field resistance is 0·08 ohm.
   (a) Calculate the torque developed at rated output.
   (b) If the maximum current on starting is not to exceed 100 A find the resistance to be connected in series at starting and the starting torque.
   (c) If a diverter of 0·16 ohm is connected in parallel across the field when the motor is running at its rated output power find the new speed. (Assume flux is proportional to field current.)

(*Answer:* 119 N.m; 2 ohms; 331 N.m; 1525 r.p.m.)

18. A 200 V, d.c. shunt motor has an armature resistance of 0·5 ohm and a field resistance of 180 ohms. The machine operates on full load at 950 r.p.m. with an armature current of 40 A. If the speed is to be decreased to 700 r.p.m., calculate the required value of series armature resistance when the load torque is proportional to speed.

(*Answer:* 1·77 ohms)

19. A 220 V, 50 h.p., d.c. shunt motor has the following open-circuit characteristics at a speed of 650 r.p.m.:

Field current (A)	0·5	1·0	1·5	2·0	2·5	3·0	3·5
Generated e.m.f. (V)	54	108	152	185	207	223	234

The armature resistance is 0·1 ohm and the shunt field resistance is 100 ohms. Calculate:

(a) The no-load speed with full field flux.
(b) The additional field resistance required for a no-load speed of 850 r.p.m.
(c) The full-load speed when the total field resistance is 110 ohms and the corresponding armature current. The fixed losses amount to 270 W.

(*Answer:* 732 r.p.m.; 29 ohms; 713 r.p.m.; 200 A)

20. A 200 V, compound d.c. motor has a total armature circuit resistance of 1·8 ohms, excluding the resistance of the series winding, the resistance of the series winding being 0·3 ohm; when the full-load armature current of 15 A flows in the series winding the proportion of series ampere-turns to shunt ampere-turns is 0·25:1. When run as a shunt machine the speed on no-load is 1600 r.p.m. Calculate the full-load speed
 (i) as a shunt motor,
 (ii) as a long shunt cumulative compound motor.

 It may be assumed that, on no-load, the armature circuit volt-drop and the series ampere-turns are both negligible. It may also be assumed that the effective flux is proportional to the field m.m.f.

(*Answer:* 1384 r.p.m.; 1080 r.p.m.)

21. Show that, when a step voltage $V\,\mathrm{H}(t)$ is applied to the armature of a separately excited d.c. motor with fixed excitation supplying a constant torque load, the energy dissipated in the armature in bringing the motor up to its steady-state speed is given by

$$\text{Energy} = \left(V + \frac{K_m T_L}{J}\right)^2 \frac{J}{2K_m^2}.$$

22. A separately excited d.c. motor with fixed excitation drives a separately excited d.c. generator with fixed excitation which is connected to a load of resistance 3.75 Ω. The armature resistance of the generator is 0·25 Ω, that of the motor is 0·5 Ω and both armature inductances can be neglected. The generated voltage constants for the motor and generator are both equal to 1 volt.sec/rad and the total moment of inertia of the rotating parts is 2 kg.m^2. If, with the system initially at rest, a step of 50 V is applied to the motor armature, find an expression for the resulting motor armature current as a function of time. Fixed losses can be neglected.

(*Answer:* $11{\cdot}1(1 + 8\,e^{-1{\cdot}13t})$)

23. A 400 V d.c. shunt motor with an armature resistance of 0·037 Ω and inductance 0·0078 H drives a constant torque load of inertia 40 kg.m^2 at a speed of 9·8 r.p.m. The torque constant is 4 N.m per ampere.

 Find the time taken for the motor to stop when it is dynamically braked with a total braking resistance of 0·5 Ω. What value of external braking resistance is required to make the motion during braking critically damped?

(*Answer:* 1·003 sec; 0·075 Ω)

24. A separately excited 200 V, d.c. motor operates at rated voltage with constant excitation on no-load. The torque constant of the motor is 4 N.m per ampere of armature current, the armature resistance is 0·5 Ω and the total moment of inertia of the rotating parts is 5 kg.m^2.

 Derive an expression, as a function of time, for the speed of the machine after plugging. Hence, or otherwise, find the time taken for the motor to stop. Fixed losses and armature inductance can be neglected.

(*Answer:* $\omega = -50 + 100\,e^{-6{\cdot}4t}$; 0·108 sec)

9. Semiconductor control of motors

The thyristor is now well established as the power control element in many forms of static electrical energy conversion system and, in particular, is widely used for the control of speed of both d.c. and a.c. motors. In the particular case of the d.c. machine, variation of the magnitude of the voltage applied to the armature and, in some cases, the field of the motor is required. When considering the control of the speed of an a.c. motor, variation in both the magnitude and frequency of the a.c. voltage applied to the motor will generally be required. These two different requirements must always be borne in mind when considering general thyristor circuits and the particular cases of a.c./d.c. conversion (controlled rectification) and d.c./a.c. conversion (controlled inversion) are the most widely used modes of operation for the control of motor speed.

9.1. The thyristor as a power device

The thyristor is a silicon semiconductor device made in a 4-layer sandwich of *pnpn* construction such that its three terminals known as the anode, cathode and gate are normally biased in the manner shown in Fig. 9.1. Since the thyristor operates in a switching mode, the power loss is inherently low.

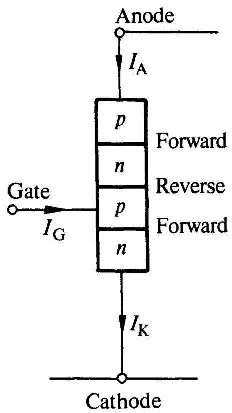


Fig. 9.1. Normal junction bias.

In practice, the losses represent only a few per cent of the maximum load power under control and, in these circumstances, the value of the load which can be controlled by a thyristor is generally limited by its voltage and current ratings, rather than the internal power loss of the device.

The thermal capacity of the semiconductor element forming a thyristor is minute and it must therefore be effectively coupled to a heat sink with large thermal capacity in order to keep the mean junction temperature to a safe value. Junction temperature affects the value of breakover voltage and current capability of the device. Excessive values of junction temperature can cause device failure and it is therefore the most important single factor controlling thyristor performance. The form of the static anode characteristic of a typical thyristor is given in Fig. 9.2.

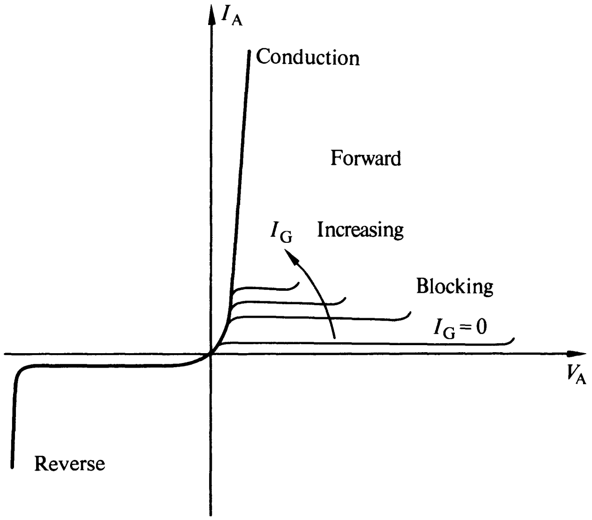


Fig. 9.2. Thyristor static anode characteristic.

9.2. Thyristor firing mechanisms

A thyristor can be turned on by an increase in temperature, by the application of gate current or by rapid change in anode voltage. Gate firing is the most useful since it utilizes the high power gain of the device and readily results in repeatable modes of operation.

When the gate current is increased, transistor action results in an increase in cathode current and firing occurs at the point of instability. The general form of the gate input characteristic is given in Fig. 9.3 and is very similar in form to the base–emitter characteristic of a transistor. Once the thyristor has fired, the gate input appears as a voltage source of the order of 0·6 V with a non-linear output resistance. This simplified treatment assumes that the thyristor is a one-dimensional device with the whole width turned on instantaneously. In practice, only a portion of the device near the gate connection is turned on instantaneously and the conducting area spreads laterally at high speed. The action is regenerative and will therefore propagate under its own influence. Initially, however, since current flow is only through a small section of the device, there will be a large local power dissipation. This effect, which is limited by the load to some extent, can be considerably reduced by using a value of gate current well above the threshold value. Normally a current pulse or pulse chain is applied to the gate and the rise-time of the pulse must be considerably less than the turn-on time of the thyristor if satisfactory performance is to be achieved.

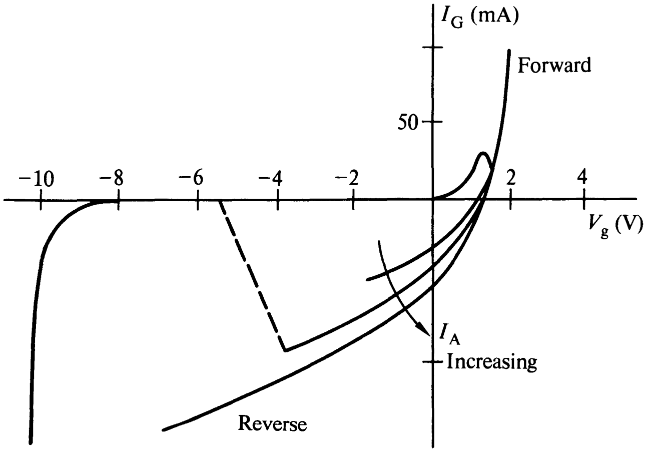


Fig. 9.3. Gate input characteristic.

A thyristor can also be fired by a rapid change in anode voltage since such a positive change results in a transient current flowing towards the gate junction. This effect momentarily increases the cathode current so that, under certain conditions, the thyristor will fire. Such an effect can be produced by supply switching surges and could result in refiring immediately after turn-off. However, susceptibility to this form of spurious firing can be reduced by including a small inductor in series with the anode and a capacitor across the thyristor. Alternatively a low resistor can be positioned between the gate and the cathode or the gate circuit can be reverse-biased immediately after gate firing has taken place.

In order to turn off a thyristor, the anode current must be reduced to a value less than the holding value and turn-off is inherently a slow process. In the conducting states, all junctions are injecting current and all the corresponding charge carriers must be removed.

The turn-off time is clearly a function of anode current and can be considerably reduced by the application of reverse anode voltage. The process of turn-off is generally known as commutation and when reverse anode voltage occurs naturally as in a.c. circuits, the process of turn-off is known as natural commutation. When, however, the supply system is d.c., no such natural reversal of anode voltage takes place and the voltage must be forced to reverse by connecting a pre-charged capacitor to the anode. Such a process is known as forced commutation and requires the use of an auxiliary circuit known as a commutation circuit. In many applications the efficiency of power conversion will be relatively low if the commutation energy in a forced commutation system is dissipated, and in these circumstances special energy recovery circuits are needed.

9.3. Gate firing

The gate characteristic of a thyristor before firing can lie between very wide limits set by temperature variations and the form of individual devices and typical extremes are given in Fig. 9.4 with limits set by maximum voltage, current and power ratings and maximum and minimum gate impedance lines. The voltage limit is virtually temperature independent but the current limit does not change with temperature. It is therefore usual to only consider the current limit for minimum temperature and also to include a lower limit in voltage below which no device will fire.

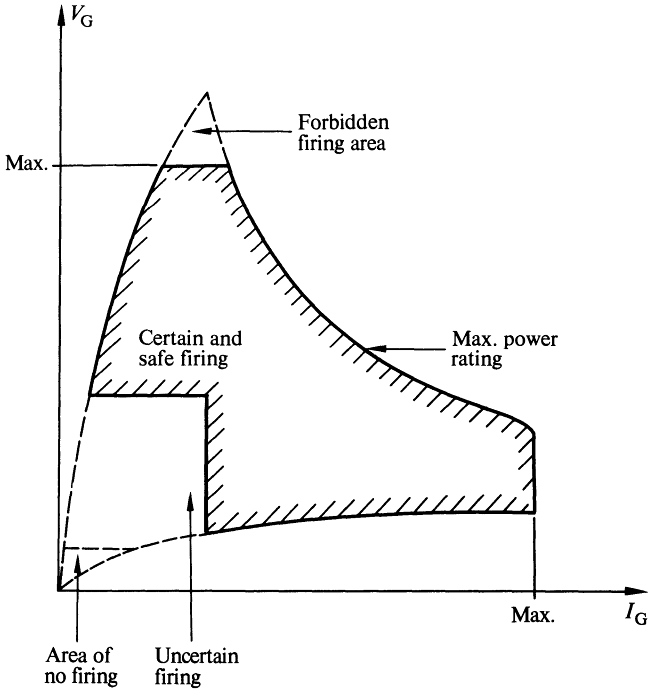


Fig. 9.4. Limits on gate characteristic for certain firing.

Any firing source can be represented by an e.m.f. E with a source resistance R_S so that:

$$I_G = \frac{E - V_G}{R_S}. \tag{9.1}$$

The graphical representation of this law is known as a load-line and the design of the firing circuit should position the load-line so that it exceeds the minimum conditions by a large margin but does not exceed any of the maximum ratings. The applied gate drive should always exceed the minimum conditions if certain and continuous firing is to be achieved. The rise-time must be shorter than the anode turn-on time and, in general, pulses shorter than 10 μs are not acceptable. In some applications, single-pulse firing of relatively short duration is possible although, in general, maintained triggering in the form of a continuous pulse or, more usually, as a pulse train consisting of short pulses (30 μs) at a high repetition rate is used.

It has previously been noted that a negative bias makes the thyristor less sensitive to unwanted turn-on and also improves the turn-off time. Small thyristors can be stabilized using an external gate–cathode resistance, but this method is not particularly useful for large devices since the gate only

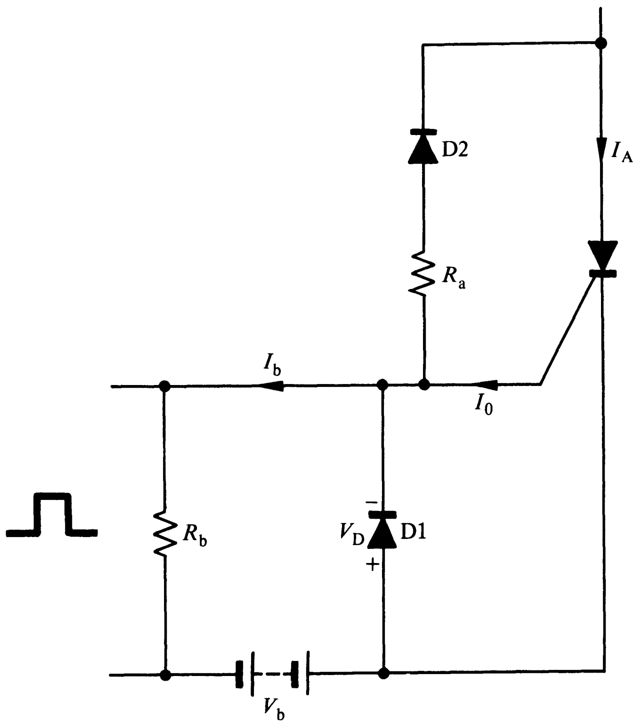


Fig. 9.5. Gate firing circuit.

directly controls that area of the junction immediately adjacent to it. A more practical method is illustrated by the circuit of Fig. 9.5. The condition required for the thyristor leakage current I_0 to pass into the gate, thereby bypassing the cathode, is that the diode D1 in Fig. 9.5 is always forward biased, i.e. $I_b > I_0$. Then approximately:

$$\frac{V_b - V_D}{R_b} > I_0. \tag{9.2}$$

The battery V_b can be replaced by a suitable Zener diode or capacitor. It is generally considered unwise to allow the gate voltage to go positive while the anode voltage is negative and this condition can be readily met by the addition of a second diode D2 and a resistor R_a to the circuit of Fig. 9.5. Operation of the firing source tends to reverse-bias diode D2 to allow the gate to go positive but the current in R_a keeps D1 conducting while the anode voltage is negative.

9.4. Series and parallel operation

In many practical circuits, it becomes necessary to operate several devices in series in order to obtain a sufficiently high safety factor for voltage transients. If a series chain of thyristors contains, say, two fast and one slow device, the full system voltage could appear across the slow thyristor for several microseconds producing very high local switching losses and this condition must be avoided. If thyristors are randomly connected in a series chain, the system voltage will divide unevenly between them according to their individual leakage characteristics. It is therefore normal practice to shunt each thyristor in a series chain with a resistor which passes a current much larger than the highest leakage current. However, such a chain of resistors is not capable of accurate voltage division at the high frequencies met in transient operation and it is usual to shunt each resistor with the capacitor to obtain proper sharing of transient over-voltage. A small resistor will normally be connected in series with each capacitor to limit the discharge current when the thyristor is fired. It is apparent that, under these conditions, the gates of the individual thyristors are at different potentials and special techniques for gate connection must be used. The most commonly used method is known as slave firing and the basis of this method is illustrated in Fig. 9.6. Thyristor TH1 is fired in the usual manner and the resulting fall in its anode voltage results in an increase in the anode–cathode voltage across thyristor TH2. A charging current then passes through the capacitor C2 and, since this current must pass through the gate circuit, thyristor TH2 will fire. This process is then repeated for all other thyristors in the series chain. The capacitor C1 in Fig. 9.6 is present to prevent spurious firing of each thyristor which may be produced by transient overvoltages. More elegant forms of this simple circuit have been in established use for many years.

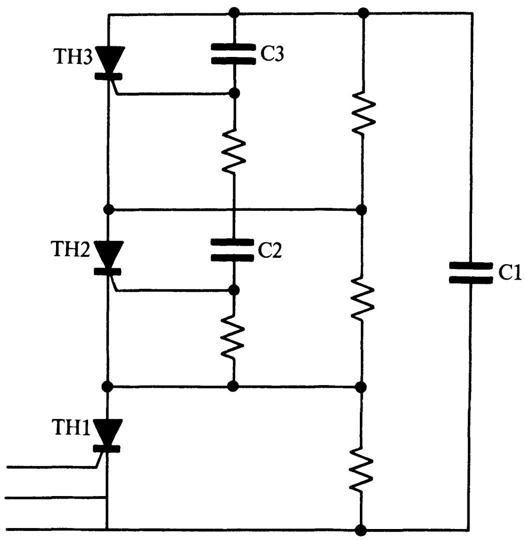


Fig. 9.6. Series operation with slave firing.

In practice, the power rating of a piece of thyristor equipment may exceed that of the biggest available device and it becomes necessary to operate thyristors is parallel. The voltage–current characteristic of a thyristor when conducting is non-linear and in these circumstances a random selection of devices operating in parallel will not share the total load in proportion to their individual ratings. If these unbalanced operating conditions are allowed to exist it is probable that either one thyristor would overheat and fail or the complete stack would be derated and some means of ensuring even current sharing must be found.

Perhaps the simplest method is to use matched devices in parallel which have been selected by the device manufacturer as having equal voltage drops on full load, but such devices are obviously expensive and may be difficult to replace in the event of a device failure. As a simple economical alternative a resistor can be inserted in series with each device in order to make the voltage drop more uniform but the extra heat to be dissipated may be a problem.

A more reliable method is to use current sharing reactors in which the out-of-balance ampere-turns induce a voltage between the anodes of the thyristors connected in parallel, thereby tending to reduce the out-of-balance currents.

There can be problems associated with the reliable firing of thyristors operating in parallel, in that the first thyristor to fire will deprive the remaining thyristors of their anode voltage. Thus some devices may not even fire and will not then take their share of the load current. The first two methods of load sharing previously mentioned, those of selected devices and series resistors, are particularly prone to this effect. The third method, that of sharing reactors, overcomes this problem since the device that fires first tends to increase the anode voltage of the other devices, thereby helping with their firing.

9.5. Overcurrent protection

Thyristors, like all semiconductor devices, have very short thermal time constants and will react to overloads through the resulting increase in junction temperature. A small increase in the temperature of the junction will produce loss of the forward voltage blocking characteristic of the device which, while not destructive in itself, could result in circuit malfunction. A large increase in junction temperature will result in loss of the reverse blocking capability and the device will be damaged, usually destroyed, by reapplication of reverse voltage. This form of failure is the most common form of failure in practice and forms the basis of overload ratings of thyristors.

Since semiconductor devices change from zero-impedance to infinite-impedance devices in a very short period of time, they have non-linear characteristics and it follows that they must be protected against high currents under short-circuit conditions and high voltage under open-circuit conditions. Any fuse used to protect a thyristor must be capable of quickly interrupting the fault current in the faulty device without producing voltages beyond the capability of other thyristors in the circuit which become effectively open-circuited when the fuse blows. For moderate overloads in a circuit operating from an a.c. supply, the fusing arc is extinguished at the first current zero after its formation and it is then possible to compare the prospective current for a given fuse pre-arcing time with the permitted device overload current for the same time period when producing a fuse rating. It is important to note that, over long time periods, the fuse characteristic is flatter than the device rating curve. In these circumstances, the fuse will not protect against long-term overloads unless the device is well-rated. In the case of a heavy overload, when the current is likely to exceed the one-cycle rating of the thyristor, the fuse must also perform a current-limiting function and manufacturers' data are generally available for the calculation of various combinations of fuse ratings and current overloads. The maximum instantaneous current that the fuse will carry must not exceed the fault current rating of the thyristor it is designed to protect, and it is usual to design the supply impedance to control the potential fault current. In many cases the arc voltage developed across a fuse will also appear across an operating thyristor and, in these circumstances, the voltage rating of the thyristor must at least equal the arc voltage. In general, fuses can be used to either protect against external faults, or perhaps of more importance, to disconnect an individual thyristor that has failed in order to maintain overall system operation.

Many practical circuits will be fitted with automatic current limiting used to satisfy two different requirements. Firstly, the load itself must be protected against high currents (for example, limiting the charge current of a battery). Secondly, the thyristors and associated semiconductor devices must be protected against sustained overloads (for example, protecting a battery charge under short-circuit conditions). Any current limiting circuit needs a current-dependent control signal which can be obtained in several different manners. The first and most simple method is to use a shunt such that the voltage developed across the shunt is amplified and then compared with a reference voltage. In d.c. circuits where electrical isolation is necessary, a d.c. current transformer can be used to replace the shunt. An a.c. current transformer can be used to provide a feedback signal from the a.c. lines to a controlled rectifier or in the output of an a.c. circuit.

9.6. Natural commutation

In any thyristor circuit operating from an a.c. supply commutation occurs naturally at voltage reversal for a resistive load and, under these circumstances, it is possible to use a thyristor as a controlled rectifier producing a d.c. voltage from an a.c. source. This configuration is illustrated in Fig. 9.7 for a resistive load. If a gate signal is applied at 150° after the start of the positive half-cycle, the thyristor will turn on and the resistor will be connected to the supply for the remaining 30° of the half-cycle. At the end of the positive half-cycle, the voltage and current will fall to zero together and the thyristor will cease to conduct. As the voltage builds up during the negative half-cycle the thyristor operates in a blocking condition and the load voltage and current will be zero during this period. Thus, as the instant in time during the positive half-cycle at which a signal is applied to the gate is varied, so the period of time for which the resistive load is connected to the supply, and hence the value of load voltage current, is varied. This process is illustrated in Fig. 9.7.

When, however, the load is a combination of inductance and resistance,

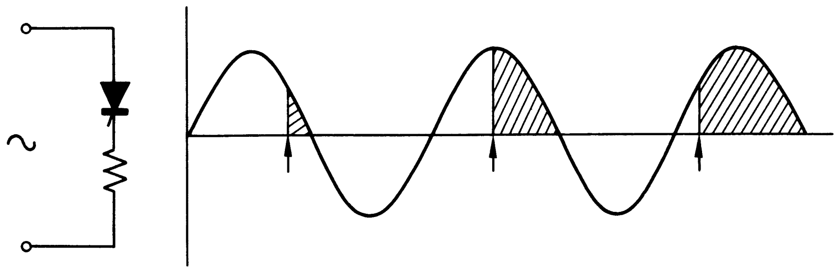


Fig. 9.7. Controlled rectification with resistive load.

the load current cannot change instantaneously. There will be an exponential change in current at the instant of firing and the load current will not be zero at the instant when the supply voltage is zero. This process is illustrated by Fig. 9.8 and it can be seen that the average value of load current is reduced by the extension of conduction into the negative half-cycle. In the extreme theoretical case of a pure inductive load, the volt-second areas above and

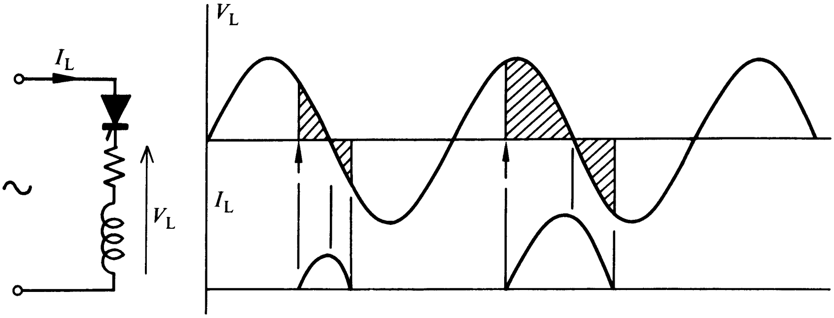


Fig. 9.8. Controlled rectification with inductive load.

below the zero voltage axis will be equal. The reduction in load voltage introduced by an inductive load can be overcome by placing a rectifier, known as a 'free-wheeling diode', across the load in a manner shown in Fig. 9.9. At the end of a positive half-cycle, the current in the load cannot be coerced to zero by the negative-going supply voltage and will continue to flow through the now low forward impedance of the free-wheeling diode.

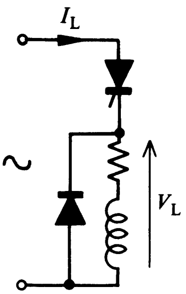


Fig. 9.9. Action of free-wheeling diode.

The current then decays at a rate governed by the time constant of the load and the forward voltage drop of the diode. In many practical cases, it will not have reached zero by the time the thyristor is refined in the next half-cycle. In these circumstances, the current is always finite. This mode of operation is known as continuous conduction and is illustrated in Fig. 9.10.

In the practical case of a thyristor circuit designed to control the armature voltage of a d.c. motor, the presence of the generated e.m.f. of the motor

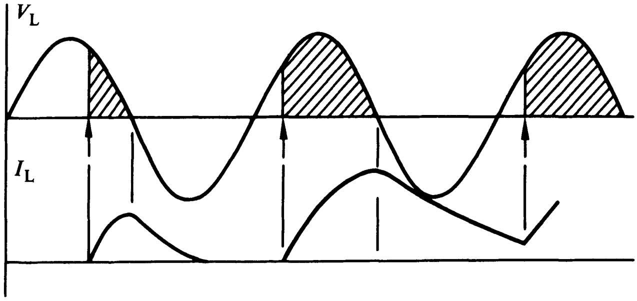


Fig. 9.10. Voltage and current waveforms with free-wheeling diode.

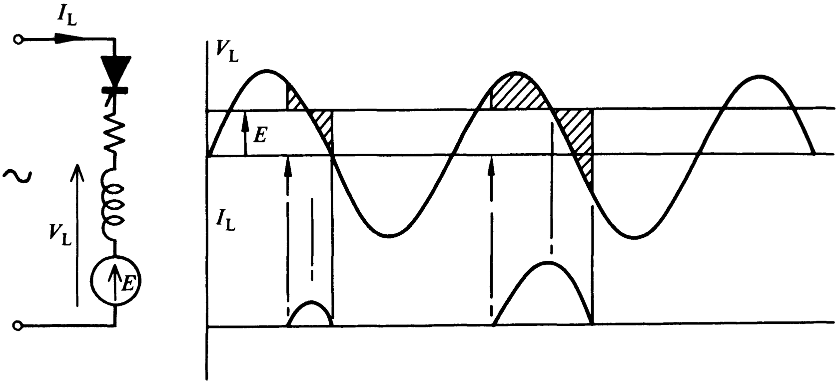


Fig. 9.11. Controlled rectification with active load.

further complicates matters. The armature circuit presents an *RL* load with a superimposed e.m.f. and the general arrangement is illustrated by Fig. 9.11 where the e.m.f. is assumed to be equal to half the peak value of the system voltage and the action of the free-wheeling diode has not been included. The supply voltage is positive with respect to the load for a range of angles between 30° and 150° after zero in the positive half-cycle and this range of angles gives the limits of control. If, for example, the thyristor is fired at 120° after zero, conduction is possible for a maximum of 60° and the current will be relatively low. As the firing angle is brought closer to zero, the conduction angle is extended and the current increases until ultimately, when the thyristor is fired at 30° after zero, the maximum conduction angle with the largest current is produced. It is apparent that the range of possible firing points for the thyristor decreases as the generated e.m.f. of the motor increases. In these circumstances, the continuous current mode of operation may not be possible for certain loading conditions even in the presence of a free-wheeling diode.

The process of natural commutation is not easy to obtain in a circuit operating from a d.c. supply, but can, however, be induced by resonant action. This method is illustrated in Fig. 9.12 which is drawn for a simple series converter. Corresponding waveforms of anode current I_A, thyristor voltage V_T and capacitor voltage V_C are given in Fig. 9.13. The thyristor is fired at $t = 0$ with zero current and, if resistance is neglected, the anode

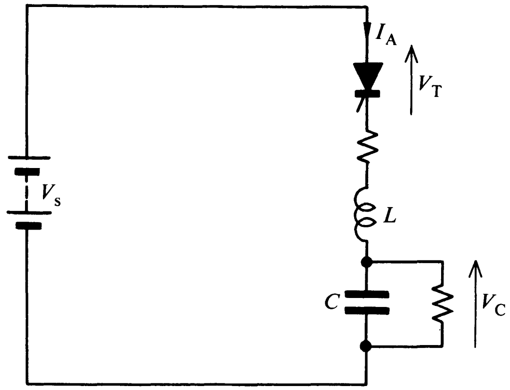


Fig. 9.12. Natural commutation with d.c. supply.

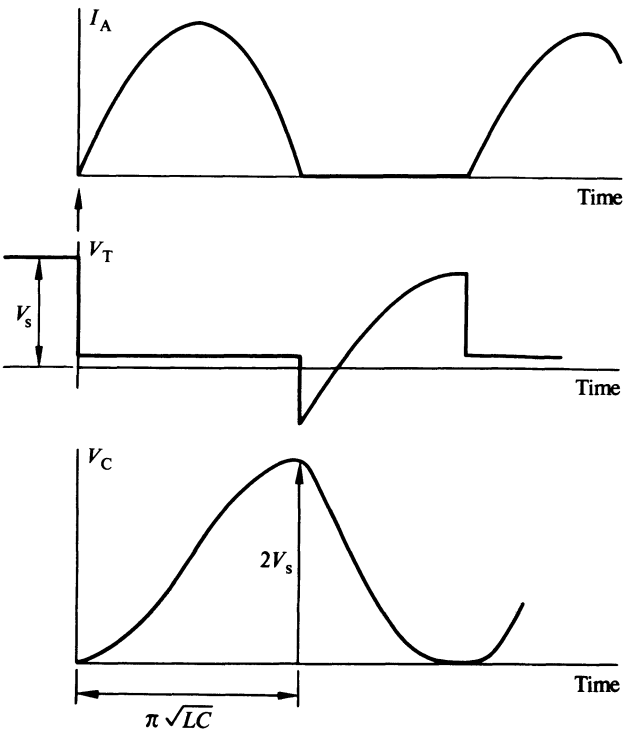


Fig. 9.13. Waveforms for natural commutation with d.c. supply.

current varies on a sinewave of half-period $\pi\sqrt{LC}$ seconds as shown. The capacitor then charges up to a voltage of $+2V_s$ (approx) and the thyristor turns off when the current through it falls below the holding value. When the thyristor has turned off, the capacitor discharges exponentially on the time constant CR to zero voltage and the cycle can be restarted. A fundamental disadvantage of this type of circuit is that the capacitor must recover to its initial state before the thyristor can be fired again.

9.7. Forced commutation

When negative anode–cathode voltage does not occur naturally in a thyristor circuit, forced commutation must be introduced. This is usually

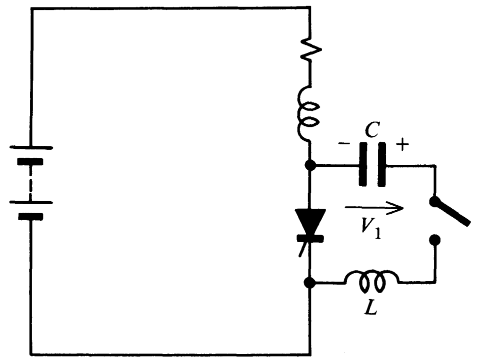


Fig. 9.14. Forced commutation with d.c. supply.

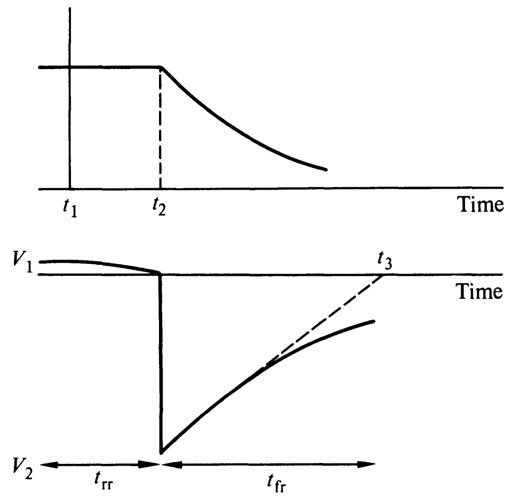


Fig. 9.15. Waveforms for forced commutation circuit.

effected by switching a charged capacitor across the conducting thyristor in order to divert current from it and, preferably, to apply reverse voltage to it for a period of time greater than its turn-off time. A simple circuit operating from a d.c. source and supplying an inductive load which utilizes forced commutation is given in Fig. 9.14 in which the capacitor C is precharged to a voltage V_1 in the sense shown. Approximate waveforms of load current I_L and thyristor voltage V_T are given in Fig. 9.15. At time t_1 the switch s (which may be an auxiliary thyristor) is closed. During the reverse recovery period $t_{rr} = t_2 - t_1$, excess charge is being flushed from the thyristor which is still presenting a low anode–cathode impedance and the load current is assumed to be constant during this period. When the thyristor begins to recover to the blocking state, the load current passes through the capacitor thereby resulting in the capacitor being recharged to the given potential of the opposite polarity. The thyristor passes reverse current during much of this time and this reverse current is limited by the presence of the inductance L_1. Positive anode voltage must not be reapplied until the thyristor has reached the forward blocking state. The total turn-off time increases slightly as the time rate of change of reapplied voltage increases and considerably with increase in junction temperature. In order to keep the total turn-off time to a reasonable value, an appreciable reverse voltage must be applied during commutation.

In practice, the main thyristor will often be shunted by a diode, such that at time t_1, with the thyristor conducting, when the switch s is closed, the diode now clamps the thyristor voltage to nearly zero as the thyristor begins to recover blocking capability. The capacitor current rises as the capacitor recharges and, ultimately, the diode becomes open-circuited. The use of this diode can shorten the effective turn-off time.

9.8. Speed control of d.c. motors

The most common requirement for a drive is that giving speed control from zero to full-load speed with a load torque which is approximately constant or increasing with speed. In such an application it is necessary to supply the armature with variable voltage, and a controlled rectification configuration operating from an a.c. source is admirably suited to this application.

The simplest and often the cheapest configuration is that of the half-wave rectifier applied to either a shunt or series d.c. motor in the manner shown in Fig. 9.16. The number of components required is minimal and, in its simplest form, it is possible to dispense with current limiting circuits. The current build-up in the positive half-cycle is then limited by the armature inductance. This system suffers from the major disadvantage that the form factor is bad (at least 2:1), leading to a significant de-rating of the motor. The torque is pulsating and this may produce objectionable results, particularly at low speeds. There will also be a large a.c. component in the armature current which can produce poor commutation. Such a system injects a d.c. component into the supply system and there will, in general, be a practical limit to the rating of such a system. In practice, this arrangement will be restricted to ratings below 1 h.p.

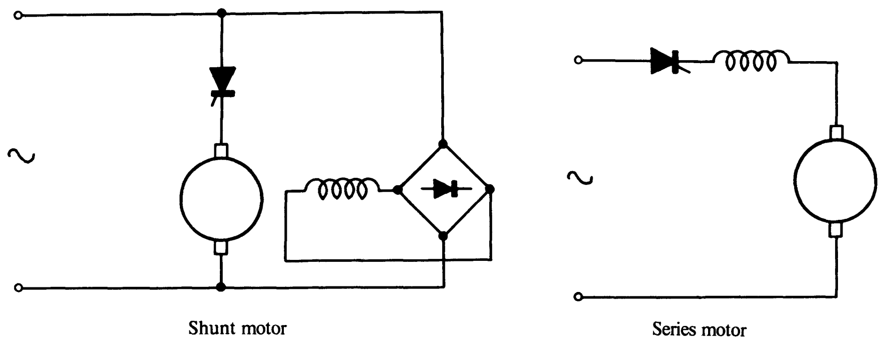


Fig. 9.16. Half-wave controller for d.c. motor.

Some improvement in form factor can be made by adding a flywheel diode across the armature. If an inductance is based in series with the armature, its effect will be to smooth out the armature current. In practice, however, a large value of inductance is necessary making such a component large and expensive. It if often cheaper to use a full-wave arrangement.

When regeneration is not needed, it is possible to use the half-controlled bridge shown in Fig. 9.17. This circuit has thyristors on two arms with cathodes common to the positive terminal and rectifiers in the other two arms. It produces full-wave rectification but has one major disadvantage in that it is essential to ensure that each thyristor is extinguished before the start of its positive half-cycle of the system voltage. This condition is important because it is possible for the armature current to flywheel through the conducting thyristor and its adjoining rectifier during the negative half-cycle. If this current is present when the system voltage becomes positive

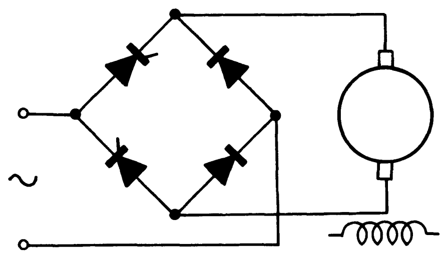


Fig. 9.17. Full-wave controller for d.c. motor.

again, a full half-cycle of the supply voltage will appear across the motor armature, resulting in a large surge of current. This effect can be prevented by either firing the second thyristor towards the end of the negative half-cycle or by using a flywheel diode across the armature so that an alternative low impedance path is provided for the current.

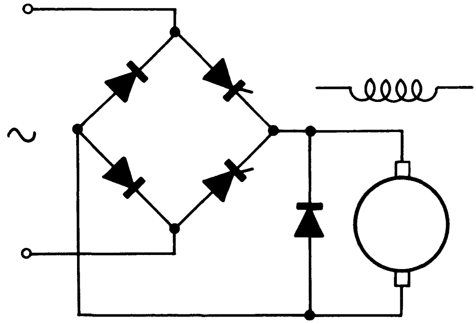


Fig. 9.18. Alternative full-wave controller.

It is, however, possible to rearrange the components in the bridge in the manner shown in Fig. 9.18, to remove the problem of failure to commutate the armature current from the conducting thyristor. In the arrangement shown in Fig. 9.18, the thyristors are commoned to a supply terminal so that the armature current circulates around the rectifiers. In these circumstances the thyristors will be extinguished by reversal of the system voltage. Thus it is possible that the current in the rectifiers will exceed the current in the thyristors and that the armature current will exceed the mains current. It then becomes necessary to use larger rectifiers or to use protective current limiting operated from the armature circuit. The circuit shown in Fig. 9.17 has the advantage that, if the rectifier bridge is completed, a supply is available for the motor field.

An alternative full-wave configuration with some useful features is shown in Fig. 9.19. This arrangement consists of a full-wave uncontrolled rectifier

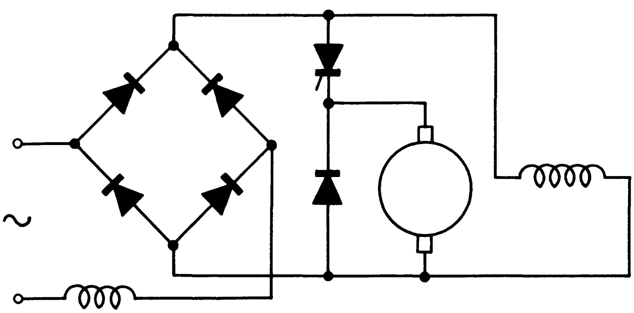


Fig. 9.19. Simple full-wave thyristor controller.

supplying a single thyristor which can now conduct on both half-cycles of the system voltage. Its original merit in the days when thyristors were very expensive was that it made full use of a single thyristor. It also has the advantage that a supply for the motor field is readily available and that, for a multi-machine system, only one main uncontrolled rectifier is necessary. Only one thyristor firing circuit is necessary and this ensures an even balance of current pulses between alternate half-cycles of the supply. Its major disadvantage is that some reliable method of extinguishing the thyristor at each zero of the mains voltage must be found so that the thyristor can enter a blocking mode to regain control during the next half-cycle.

All the circuits so far described have operated from a single-phase supply with either line to neutral or line to line voltage. The normal acceptable loading limit for a single-phase system is about 7 kW and for ratings above this, it is necessary to use a 3-phase supply with the corresponding thyristor configuration.

The simplest 3-phase arrangement is the half-controlled bridge with thyristors in one half and rectifiers in the other half of the bridge, connected in the manner shown in Fig. 9.20. Such an arrangement suffers from the same commutation trouble as the single-phase bridge. In these circumstances either a minimum firing angle unit must be used or a flywheel diode must be fitted across the armature.

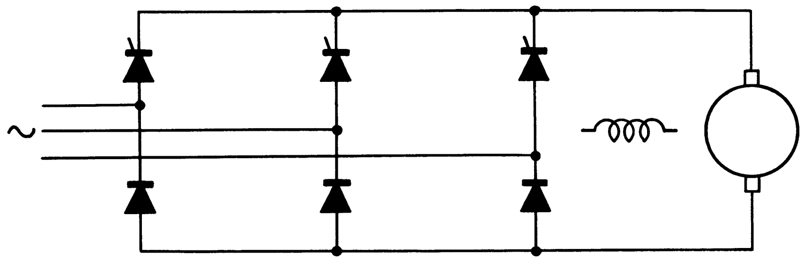


Fig. 9.20. Half-wave three-phase controller.

In some applications, a d.c. motor will be used to speed control an overhauling load such that, under certain circumstances, power transfer from the load to the motor is possible. Under these conditions the motor will act as a generator and a thyristor power converter capable of transferring power into the a.c. mains must be used. The basic circuit of such a regenerative system is shown in Fig. 9.21. Under regenerative conditions current must flow out of the positive motor terminal into the supply system and the thyristor TH1 in Fig. 9.21 controls the amount of regenerative power returned to the mains while TH2 controls the system under motoring conditions. Care must obviously be taken to ensure that both thyristors are not conducting at the same time and, as previously noted, a thyristor can only be turned off by having its current reduced to below the holding value. In a regenerative

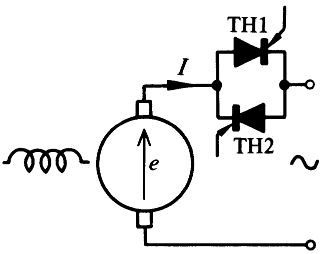


Fig. 9.21. Regenerative controller for d.c. motor.

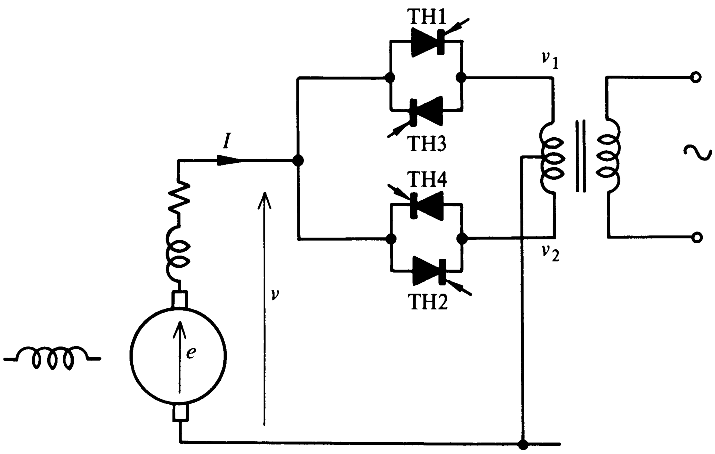


Fig. 9.22. Bi-phase regenerative controller.

circuit of this type it is usual to ensure that the net circuit e.m.f. always acts in a direction such that the rate of change of current is negative and to ensure that the e.m.f. is present until zero current is reached.

This simple circuit suffers from several major disadvantages and, in particular, a severe restriction must be placed on the value of the firing angle during regeneration if uncontrolled conduction is to be avoided. This type of restriction can be overcome by the use of the bi-phase, half-wave system illustrated in Fig. 9.22, for which continuous conduction is possible with the current being forcibly transferred from one thyristor to the other because the cathode voltage of the incoming device is lower than that of the outgoing device at the instant of firing.

A fuller treatment of more complex multi-phase thyristor circuits, including regenerative controllers can be found in ref. 1.

9.9. Speed control of a.c. motors

Most a.c. motors operate at constant speed and speed control can be obtained by varying the frequency of the applied voltage. In many cases the magnitude of the applied voltage will also be varied in direct proportion to the frequency in order to maintain the flux in the machine at a constant value. In general, a static power converter producing a variable frequency, variable magnitude polyphase output voltage from fixed polyphase a.c. mains is required and this can be achieved in one of two ways. Firstly, a direct conversion (a.c. to a.c.) can be made using the so-called cycloconverter principle and a full discussion of such circuits can be found in ref. 1. The second and more common way is to convert the fixed a.c. to variable d.c. and to then reconvert this d.c. voltage to the required variable a.c. system; such a method uses a d.c. link. In this case negative anode–cathode voltage does not occur naturally in the d.c. to a.c. thyristor converter and the process of forced commutation must be used. Two basic methods of forced commutation inverters, the so-called parallel inverter and the pulse-width modulated inverter are in common use.

A simplified diagram representing a single-phase parallel inverter is given in Fig. 9.23 in which the inductance L between the source and the thyristors

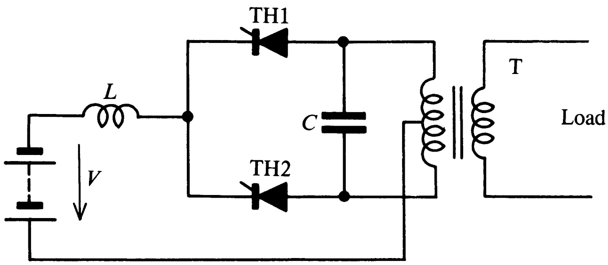


Fig. 9.23. Single-phase parallel inverter.

acts as a current limiter. With thyristor TH1 in Fig. 9.23 conducting, the supply voltage V appears across one half of the primary of the output transformer T and load current flows. The voltage across the whole of the transformer primary winding is then $2V$ and the capacitor C is charged to the voltage $2V$. When thyristor TH2 is fired, the capacitor discharges through the two thyristors and TH1 is reverse biased until it turns off. Thyristor TH2 is then in a conducting state and the supply voltage V appears across the other half of the transformer primary in an opposite sense. The output voltage across the secondary then reverses and is therefore an approximate square wave whose frequency is controlled by the firing pulses applied to the two thyristors. Filtering can be introduced at the output if a sinusoidal output voltage is required.

The general principles of pulse-width modulation are illustrated by the circuit of Fig. 9.24. Thyristor TH1 is the main circuit device and thyristor TH2, capacitor C and resistor R form the commutating circuit for the main thyristor. The presence of the diode D is now essential in that it provides a path for load current and allows the capacitor to discharge. Initially neither thyristor is conducting so that both points A and B in Fig. 9.24 are at the potential of the negative rail. If TH1 is fired the point B in Fig. 9.24 will now be at the potential of the positive rail. Current will build up in the load on an exponential of time constant L/r and, at the same time, the capacitor C will be charged to a voltage $+V$, through the resistor R, with the potential at point B positive with respect to that at point A. If after some convenient time, thyristor TH2 is fired, the potential at point A will rise to $+V$ volts, the presence of the charged capacitor will result in the potential at point B, in Fig. 9.24, rising to $+2V$ volts. The main thyristor TH1 will then be reverse

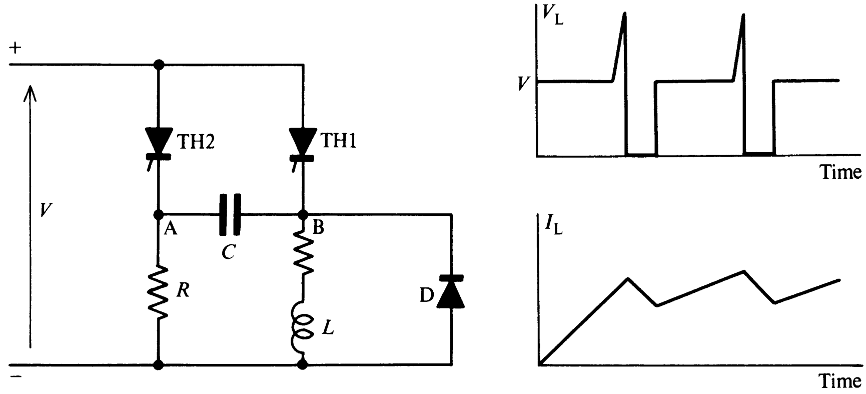


Fig. 9.24. Pulse-width modulation controller.

biased and can turn off. Load current must still be flowing, because of the presence of load inductance and this current is supplied by the charge stored in the capacitor. The capacitor will thus be discharged resonantly until point B goes to a negative potential, at which point the diode D will conduct, thereby clamping the load voltage at approximately zero and also providing a flywheel path for the load current. Capacitor C will now have a voltage of $+V$ across it with point A positive and the thyristor TH1 will be conducting with a small current passing through the resistor R. The load current will fall exponentially until thyristor TH2 is fired again. This will cause the potential at point B to rise to $+V$ so that the potential at point A rises to $+2V$. The thyristor TH1 is thus extinguished and the capacitor C recharges through R with the point B again at the positive potential, ready for the next cycle of operation.

More elegant forms of the basic circuit of Fig. 9.24 can be used to generate a sinusoid of output voltage with a superimposed high frequency ripple. The ripple frequency is controlled by the maximum permissible switching rate for the thyristors and the allowable commutation loss which takes place as switching occurs. The frequency of the output voltage is readily controlled by control of the pulse repetition rate of the gate signal.

A full treatment of semiconductor control of a.c. machines is given in ref. 2.

References

1. Pelly, B. R., *Thyristor Phase-controlled Convertors and Cycloconverters*, Wiley, 1971.
2. Murphy, J. M. D., *Thyristor Control of Alternating Current Motors*, Pergamon, 1973.

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